

**GREEN'S RELATIONS AND THEIR
GENERALIZATIONS ON SEMIGROUPS**

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Dedicated to the 65th birthday of Klaus Denecke

Abstract

Green's relations and their generalizations on semigroups are useful in studying regular semigroups and their generalizations. In this paper, we first give a brief survey of this topic. We then give some examples to illustrate some special properties of generalized Green's relations which are related to completely regular semigroups and abundant semigroups.

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