

*Discussiones Mathematicae*  
*Probability and Statistics 29 (2009) 185–197*  
doi:10.7151/dmps.1114

**GENERALIZED  $F$  TESTS IN MODELS  
WITH RANDOM PERTURBATIONS:  
THE GAMMA CASE**

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**Abstract**

Generalized  $F$  tests were introduced for linear models by Michal-ski and Zmysłony (1996, 1999). When the observations are taken in not perfectly standardized conditions the  $F$  tests have generalized  $F$  distributions with random non-centrality parameters, see Nunes and Mexia (2006). We now study the case of nearly normal perturbations leading to Gamma distributed non-centrality parameters.

**Keywords:** generalized  $F$  distributions; random non-centrality parameters; Gamma distribution.

**2000 Mathematics Subject Classification:** 62J12, 62H10, 62J99.

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Received 10 November 2009