

Discussiones Mathematicae
Probability and Statistics 29 (2009) 131–153
doi:10.7151/dmps.1111

**SOME APPLICATIONS OF PROBABILITY
GENERATING FUNCTION BASED METHODS
TO STATISTICAL ESTIMATION**

MANUEL L. ESQUÍVEL

Departamento de Matemática
Faculdade de Ciências e Tecnologia
Universidade Nova de Lisboa, Portugal

e-mail: mle@fct.unl.pt

This work is dedicated to my dearest friend and colleague João Tiago Mexia, as a token of everlasting admiration, respect and gratitude, being certain that his unbreakable enthusiasm for Mathematics and mathematicians will always be, for us, a source of inspiration and guidance.

Abstract

After recalling previous work on probability generating functions for real valued random variables we extend to these random variables uniform laws of large numbers and functional limit theorem for the empirical probability generating function. We present an application to the study of continuous laws, namely, estimation of parameters of Gaussian, gamma and uniform laws by means of a minimum contrast estimator that uses the empirical probability generating function of the sample. We test the procedure by simulation and we prove the consistency of the estimator.

Keywords: probability generating function, empirical laws, estimation of parameters of continuous laws.

2000 Mathematics Subject Classification: 60–08, 60E10, 30B50.

REFERENCES

- [1] D. Dacunha-Castelle and M. Duflo, *Probabilités et Statistiques*, Tome 2, Problèmes à temps mobile, Masson, Paris 1983.
- [2] M.M. Dowling and M. Nakamura, *Estimating parameters for discrete distributions via the empirical probability generating function*, Commun. Stat., Simulation Comput **26** (1) (1997), 301–313.
- [3] M.L. Esquivel, *Probability generating functions for discrete real-valued random variables*, Teor. Veroyatn. Primen. **52** (1) (2007), 129–149; translation in Theory Probab. Appl. **52** (1) (2008), 40–57.
- [4] M.L. Esquivel, *Aplicações das funções geradoras de probabilidade a variáveis aleatórias reais*, *Proceedings of the XII Annual Congress Portuguese Statistical Society*, Editors Carlos Braumann, Paulo Infante, Manuela Oliveira, Russell Alpizar Jara e Fernando Rosado (2005), 235–246.
- [5] A. Feuerverger, *On the empirical saddlepoint approximation*, Biometrika **76** (3) (1989), 457–464.
- [6] A. Feuerverger and P. McDunnough, *On statistical transform methods and their efficiency*, The Can. J. of Stat. **12** (4) (1984), 303–317.
- [7] P. Gaenssler, *Empirical Processes*, Lecture Notes-Monograph Series, volume 3, Institute of Mathematical Statistics, Hayward, CA 1983.
- [8] S. Kocherlakota and K. Kocherlakota, *Goodness of fit tests for discrete distributions*, Commun. Statist.-Theor. Meth. **15** (3) (1986), 815–829.
- [9] P. Malliavin, *Integration and Probability*, Springer Verlag 1995.
- [10] M.S. Marques and V. Pérez-Abreu, *Law of large numbers and central limit theorem for the empirical probability generating function of stationary random sequences and processes*, Aportaciones Mat., Notas Invest. **4** (2) (1989), 100–109.
- [11] M. Nakamura and V. Pérez-Abreu, *Empirical probability generating function. An overview*, Insur. Math. Econ. **12** (3) (1993), 349–366.
- [12] M. Nakamura and V. Pérez-Abreu, *Exploratory data analysis for counts using the empirical probability generating function*, Commun. Stat., Theory Methods **22** (3) (1993), 827–842.
- [13] M. Nakamura and V. Pérez-Abreu, *Use of an empirical probability generating function for testing a Poisson model*, Can. J. Stat. **21** (2) (1993), 149–156.
- [14] W.R. Pestman, *Mathematical Statistics*, Walter de Gruyter, Paris, New York 1998.

[15] B.L.S. Prakasa Rao, *Statistical Inference for Diffusion Type Processes*, Arnold Hodder Headline Group 1999.

[16] B. Rémillard and R. Theodorescu, *Inference based on the empirical probability generating function for mixtures of Poisson distributions*, Stat. Decis. **18** (4) (2000), 349–366.

- [17] R. Rueda and F. O'Reilly, *Tests of fit for discrete distributions based on the probability generating function*, Commun. Stat., Simulation Comput. **28** (1) (1999), 259–274.
- [18] R. Rueda, V. Pérez-Abreu and F. O'Reilly, *Goodness of fit for the Poisson distribution based on the probability generating function*, Commun. Stat., Theory Methods **20** (10) (1991), 3093–3110.

Received 11 October 2009