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## INFERENCE ON THE LOCATION PARAMETER OF EXPONENTIAL POPULATIONS

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*A token of friendship to Professor J. T. Mexia  
on his 70th birthday*

### Abstract

Studentization and analysis of variance are simple in Gaussian families because  $\bar{X}$  and  $S^2$  are independent random variables. We exploit the independence of the spacings in exponential populations with location  $\lambda$  and scale  $\delta$  to develop simple ways of dealing with inference on the location parameter, namely by developing an analysis of scale in the homocedastic independent  $k$ -sample problem.

**Keywords:** studentization, analysis of scale, characterizations, independence of exponential spacings, location-scale families,  $F$ -ratio.

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