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BAYESIAN AND GENERALIZED CONFIDENCE INTERVALS ON VARIANCE RATIO AND ON THE VARIANCE COMPONENT IN MIXED LINEAR MODELS

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Abstract

The paper deals with construction of exact confidence intervals for the variance component σ_1^2 and ratio θ of variance components σ_1^2 and σ^2 in mixed linear models for the family of normal distributions $\mathcal{N}_t(0, \quad \sigma_1^2 W + \sigma^2 I_t)$. This problem essentially depends on algebraic structure of the covariance matrix W (see Gnot and Michalski, 1994, Michalski and Zmyślony, 1996). In the paper we give two classes of bayesian interval estimators depending on a prior distribution on $(\sigma_1^2 \ , \sigma^2)$ for:

1) the variance components ratio θ - built by using test statistics obtained from the decomposition of a quadratic form $y'Ay$ for the Bayes locally best estimator of σ_1^2, Michalski and Zmyślony (1996),

2) the variance component σ_1^2 - constructed using Bayes point estimators from BIQUE class (Best Invariant Quadratic Unbiased Estimators, see Gnot and Kleffe, 1983, and Michalski, 2003).

In the paper an idea of construction of confidence intervals using generalized p-values is also presented (Tsui and Weerahandi, 1989, Zhou and Mathew, 1994). Theoretical results for Bayes interval estimators and for some generalized confidence intervals by simulations studies for some experimental layouts are illustrated and compared (cf Arendacká, 2005).

Keywords: mixed linear models; variance components; hypothesis testing; confidence intervals; generalized p-values.

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