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ON SOME LIMIT DISTRIBUTIONS FOR GEOMETRIC RANDOM SUMS

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Abstract

We define and give the various characterizations of a new subclass of geometrically infinitely divisible random variables. This subclass, called geometrically semistable, is given as the set of all these random variables which are the limits in distribution of geometric, weighted and shifted random sums. Introduced class is the extension of, considered until now, classes of geometrically stable [5] and geometrically strictly semistable random variables [10]. All the results can be straightforward transfered to the case of random vectors in $\mathbb{R}^d$.

Keywords: random sum, infinite divisibility, semistability, geometric infinite divisibility, geometric stability, geometric semistability, characteristic function, limit distribution, Lévy process.

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