

STOCHASTIC VORTICES IN PERIODICALLY RECLASSIFIED POPULATIONS

GRACINDA RITA GUERREIRO

AND

JOÃO TIAGO MEXIA

Department of Mathematics, FCT - New University of Lisbon
Campus da Caparica, 2829-516 Caparica, Portugal

e-mail: grg@fct.unl.pt

Abstract

Our paper considers open populations with arrivals and departures whose elements are subject to periodic reclassifications. These populations will be divided into a finite number of sub-populations.

Assuming that:

a) entries, reclassifications and departures occur at the beginning of the time units;

b) elements are reallocated at equally spaced times;

c) numbers of new elements entering at the beginning of the time units are realizations of independent Poisson distributed random variables;

we use Markov chains to obtain limit results for the relative sizes of the sub-populations corresponding to the states of the chain. Namely we will obtain conditions for stability of the relative sizes for transient and recurrent states as well as for all states. The existence of such stability corresponds to the existence of a stochastic structure based either on the transient or on the recurrent states or even on all states. We call these structures stochastic vortices because the structure is maintained despite entrances, departures and reallocations.

Keywords: Markov chains, stochastic vortices.

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