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CERTAIN NEW M-MATRICES AND THEIR PROPERTIES WITH APPLICATIONS

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Abstract

The M_n-matrix was defined by Mohan [21] who has shown a method of constructing $(1, -1)$-matrices and studied some of their properties. The $(1, -1)$-matrices were constructed and studied by Cohn [6], Ehrlich [9], Ehrlich and Zeller [10], and Wang [34]. But in this paper, while giving some resemblances of this matrix with a Hadamard matrix, and by naming it as an M-matrix, we show how to construct partially balanced incomplete block designs and some regular graphs by it. Two

types of these M-matrices have been considered. Also we will make a mention of certain applications of these M-matrices in signal and communication processing, and network systems and end with some open problems.

Keywords: M-matrices, non-orthogonality, orthogonal number, Hadamard matrix, partially balanced incomplete block (PBIB) design, regular graph.

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