

## ON COVARIETY LATTICES

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### Abstract

This paper shows basic properties of covariety lattices. Such lattices are shown to be infinitely distributive. The covariety lattice  $L_{\mathcal{CV}}(\mathbf{K})$  of subcovarieties of a covariety  $\mathbf{K}$  of  $F$ -coalgebras, where  $F : \mathbf{Set} \rightarrow \mathbf{Set}$  preserves arbitrary intersections is isomorphic to the lattice of subcoalgebras of a  $\mathcal{P}_\kappa$ -coalgebra for some cardinal  $\kappa$ . A full description of the covariety lattice of  $\mathcal{I}d$ -coalgebras is given. For any topology  $\tau$  there exist a bounded functor  $F : \mathbf{Set} \rightarrow \mathbf{Set}$  and a covariety  $\mathbf{K}$  of  $F$ -coalgebras, such that  $L_{\mathcal{CV}}(\mathbf{K})$  is isomorphic to the lattice  $(\tau, \cup, \cap)$  of open sets of  $\tau$ .

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### REFERENCES

- [1] M. Barr, *Terminal Coalgebras in Well-founded Set Theory*, Theoretical Computer Science **144** (2) (1993), 299–315.
- [2] H.P. Gumm, *Elements of the General Theory of Coalgebras*, LUATCS'99, Rand Afrikaans University, Johannesburg, South Africa 1999.

- [3] H.P. Gumm, *Functors for coalgebras*, Algebra Universalis **45** (2–3) (2001), 135–147.
- [4] H.P. Gumm and T. Schröder, *Coalgebras of bounded type*, Mathematical Structures in Computer Science **12** (5) (2002), 565–578.
- [5] H.P. Gumm, *From  $T$ -coalgebras to filter structures and transition systems*, CALCO 2005, Springer Lecture Notes in Computer Science (LNCS) 3629, 2005.

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