

OPTIMAL CONTROL OF SYSTEMS DETERMINED
BY STRONGLY NONLINEAR OPERATOR
VALUED MEASURES

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Abstract

In this paper we consider a class of distributed parameter systems (partial differential equations) determined by strongly nonlinear operator valued measures in the setting of the Gelfand triple $V \hookrightarrow H \hookrightarrow V^*$ with continuous and dense embeddings where H is a separable Hilbert space and V is a reflexive Banach space with dual V^* . The system is given by

$$dx + A(dt, x) = f(t, x)\gamma(dt) + B(t)u(dt), \quad x(0) = \xi, \quad t \in I \equiv [0, T]$$

where A is a strongly nonlinear operator valued measure mapping $\Sigma \times V$ to V^* with Σ denoting the sigma algebra of subsets of the set I and f is a nonlinear operator mapping $I \times H$ to H , γ is a countably additive bounded positive measure and the control u is a suitable vector measure. We present existence, uniqueness and regularity properties of weak solutions and then prove the existence of optimal controls (vector valued measures) for a class of control problems.

Keywords: evolution equations, strongly nonlinear operator valued measures, existence of solutions, regularity properties, optimal control.

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REFERENCES

- [1] N.U. Ahmed, *Differential inclusions, operator valued measures and optimal control*, Special Issue of Dynamic Systems and Applications, Set-Valued Methods in Dynamic Systems, Guest Editors: M. Michta and J. Motyl **16** (2007), 13–36.
- [2] N.U. Ahmed, *Evolution equations determined by operator valued measures and optimal control*, Nonlinear Analysis **67** (2007), 3199–3216.
- [3] N.U. Ahmed, *Vector and operator valued measures as controls for infinite dimensional systems: optimal control*, Discuss. Math. Differential Inclusions Control and Optimization **28** (2008), 95–131.
- [4] N.U. Ahmed, *A class of semilinear parabolic and hyperbolic systems determined by operator valued measures*, DCDIS **14** (4) (2007).
- [5] N.U. Ahmed, *Parabolic systems determined by strongly nonlinear operator valued measures*, Nonlinear Analysis, Special Issue (Felicitation of Professor V. Lakshmikantham on his 85th birth date).
- [6] N.U. Ahmed, *Optimization and Identification of Systems Governed by Evolution Equations on Banach Spaces*, Pitman Research Notes in Mathematics Series 184, Longman Scientific and Technical, U.K. and co-publisher John Wiley, New York, 1988.
- [7] N.U. Ahmed, K.L. Teo and S.H. Hou, *Nonlinear impulsive systems on infinite dimensional spaces*, Nonlinear Analysis **54** (2003), 907–925.
- [8] N.U. Ahmed, *Some remarks on the dynamics of impulsive systems in Banach spaces*, DCDIS **8** (2001), 261–274.
- [9] N.U. Ahmed, *Impulsive perturbation of C_0 -semigroups by operator valued measures*, Nonlinear Functional Analysis & Applications **9** (1) (2004), 127–147.
- [10] N.U. Ahmed, *Existence of optimal controls for a general class of impulsive systems on Banach spaces*, SIAM J. Control. Optim. **42** (2) (2003), 669–685.
- [11] J. Diestel and J.J. Uhl, Jr., *Vector Measures*, Mathematical Surveys, no. 15, American Mathematical Society, Providence, Rhode Island, 1977.
- [12] N. Dunford and J.T. Schwartz, *Linear Operators, Part 1: General Theory*, Interscience Publishers, Inc., New York, London, 1958, 1964.
- [13] H.O. Fattorini, *Infinite Dimensional Optimization and Control Theory*, Encyclopedia of Mathematics and its Applications, Vol. 62, Cambridge University, Cambridge, U.K., 1999.

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