

## BOUNDARY VALUE PROBLEMS FOR DIFFERENTIAL INCLUSIONS WITH FRACTIONAL ORDER

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### Abstract

In this paper, we shall establish sufficient conditions for the existence of solutions for a boundary value problem for fractional differential inclusions. Both cases of convex valued and nonconvex valued right hand sides are considered.

**Keywords and phrases:** differential inclusion, Caputo fractional derivative, fractional integral, existence, fixed point.

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