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VECTOR AND OPERATOR VALUED MEASURES AS CONTROLS FOR INFINITE DIMENSIONAL SYSTEMS: OPTIMAL CONTROL

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Abstract

In this paper we consider a general class of systems determined by operator valued measures which are assumed to be countably additive in the strong operator topology. This replaces our previous assumption of countable additivity in the uniform operator topology by the weaker assumption. Under the relaxed assumption plus an additional assumption requiring the existence of a dominating measure, we prove some results on existence of solutions and their regularity properties both for linear and semilinear systems. Also presented are results on continuous dependence of solutions on operator and vector valued measures, and other parameters determining the system which are then used to prove some results on control theory including existence and necessary conditions of optimality. Here the operator valued measures are treated as structural controls. The paper is concluded with some examples from classical and quantum mechanics and a remark on future direction.

Keywords: evolution equations, Banach spaces, operator valued measures, strong operator topology, existence of solutions, optimal control.

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