

**SET-VALUED FRACTIONAL ORDER  
DIFFERENTIAL EQUATIONS IN THE SPACE  
OF SUMMABLE FUNCTIONS**

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**Abstract**

In this paper, we study the existence of integrable solutions for the set-valued differential equation of fractional type

$$\left( D^{\alpha_n} - \sum_{i=1}^{n-1} a_i D^{\alpha_i} \right) x(t) \in F(t, x(\varphi(t))),$$

a.e. on  $(0, 1)$ ,  $I^{1-\alpha_n} x(0) = c$ ,  $\alpha_n \in (0, 1)$ ,

where  $F(t, \cdot)$  is lower semicontinuous from  $\mathbb{R}$  into  $\mathbb{R}$  and  $F(\cdot, \cdot)$  is measurable. The corresponding single-valued problem will be considered first.

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