

## ASYMPTOTIC BEHAVIOUR OF SOLUTIONS OF DIFFERENCE EQUATIONS IN BANACH SPACES

ANNA KISIOLEK

*Technical University of Poznań*

*Piotrowo 3, PL-60-965 Poznań, Poland*

**e-mail:** akisiolek@wp.pl

### Abstract

In this paper we consider the first order difference equation in a Banach space

$$\Delta x_n = \sum_{i=0}^{\infty} a_n^i f(x_{n+i}).$$

We show that this equation has a solution asymptotically equal to  $a$ .

As an application of our result we study the difference equation

$$\Delta x_n = \sum_{i=0}^{\infty} a_n^i g(x_{n+i}) + \sum_{i=0}^{\infty} b_n^i h(x_{n+i}) + y_n$$

and give conditions when this equation has solutions.

In this note we extend the results from [8, 9]. For example, in [9] the function  $f$  is a real Lipschitz function. We suppose that  $f$  has values in a Banach space and satisfies some conditions with respect to the measure of noncompactness and measure of weak noncompactness.

**Keywords:** Banach space, difference equation, fixed point, measure of noncompactness, asymptotic behaviour of solutions.

**2000 Mathematics Subject Classification:** 39A10, 47N99.

## REFERENCES

- [1] O. Arino, S. Gautier and J.P. Penot, *A fixed point theorem for sequentially continuous mappings with application to ordinary differential equations*, *Func. Ekvac.* **27** (1984), 273–279.
- [2] J.M. Ball, *Properties of mappings and semigroups*, *Proc. Royal. Soc. Edinburg Sect. (A)* **72** (1973/74), 275–280.
- [3] J. Banaś and K. Goebel, *Measures of noncompactness in Banach spaces*, *Lecture Notes in Pure and Applied Mathematics*, **60**, Marcel Dekker, New York-Basel, 1980.
- [4] J. Banaś and J. Rivero, *Measures of weak noncompactness*, *Ann. Math. Pura Appl.* **125** (1987), 213–224.
- [5] G. Darbo, *Punti uniti in trasformazioni a codominio non compatto*, *Rend. Sem. Mat. Univ. Padova* **24** (1955), 84–92.
- [6] M. Dawidowski, I. Kubiacyk and J. Morchała, *A discrete boundary value problem in Banach spaces*, *Glasnik Mathematicki*, **36** 56(2001), 233–239.
- [7] F.S. de Blasi, *On a property of the unit sphere in Banach space*, *Bull. Math. Soc. Sci. Math. R.S. Raumannie* **21** (1997), 259–262.
- [8] C. Gonzalez and A. Jimenez-Melado, *An application of Krasnoselskii fixed point theorem to the asymptotic behavior of solutions of difference equations in Banach spaces*, *J. Math. Anal. Appl.* **247** (2000), 290–299.
- [9] C. Gonzalez and A. Jimenez-Melado, *Asymptotic behaviour of solutions of difference equations in Banach spaces*, *Proc. Amer. Math. Soc.*, **128** (6) (2000), 1743–1749.
- [10] I. Kubiacyk, *On a fixed point theorem for weakly sequentially continuous mapping*, *Discuss. Math. Diff. Incl.* **15** (1995), 15–20.
- [11] A.R. Mitchell and C. Smith, *An existence theorem for weak solutions of differential equations in Banach spaces*, *Nonlinear equations in abstract spaces*, V. Lakshmikantham, ed. 387–404, Orlando, 1978.

Received 10 October 2003

Revised 3 January 2005