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SOME REMARKS ON OPERATORS PRESERVING PARTIAL ORDERS OF MATRICES

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Abstract

Stępniaak [Linear Algebra Appl. 151 (1991)] considered the problem of equivalence of the Löwner partial order of nonnegative definite matrices and the Löwner partial order of squares of those matrices. The paper was an important starting point for investigations of the problem of how an order between two matrices \mathbf{A} and \mathbf{B} from different sets of matrices can be preserved for the squares of the corresponding matrices \mathbf{A}^2 and \mathbf{B}^2 , in the sense of the Löwner partial ordering, the star partial ordering, the minus partial ordering, and the sharp partial ordering. Many papers have since been published (mostly coauthored by **J.K. Baksalary - to whom the present paper is dedicated**) that generalize the results in two directions: by widening the class of matrices considered and by replacing the squares by arbitrary powers. In the present paper we make a résumé of some of these results and suggest some further generalizations for polynomials of the matrices considered.

Keywords: star partial ordering, minus partial ordering, löwner partial ordering, sharp partial order, nonnegative definite matrix, Hermitian matrix, EP-matrix, power of a matrix, polynomial of a matrix.

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