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**AN ALTERNATIVE APPROACH TO
CHARACTERIZE THE COMMUTATIVITY
OF ORTHOGONAL PROJECTORS**

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Abstract

In an invited paper, Baksalary [Algebraic characterizations and statistical implications of the commutativity of orthogonal projectors. In: T. Pukkila, S. Puntanen (Eds.), Proceedings of the Second International Tampere Conference in Statistics, University of Tampere, Tampere, Finland, [2], pp. 113–142] presented 45 necessary and sufficient conditions for the commutativity of a pair of orthogonal projectors. Basing on these results, he discussed therein also statistical aspects of the commutativity with reference to problems concerned with canonical correlations and with comparisons between estimators and between sets of linearly sufficient statistics corresponding to different linear models. In the present paper, parts of this analysis are resumed in order to shed some additional light on the problem of commutativity. The approach utilized is different than the one used by Baksalary, and is based on representations of projectors in

terms of partitioned matrices. The usefulness of such representations is demonstrated by reinvestigating some of Baksalary's statistical considerations.

Keywords: partitioned matrix, canonical correlations, ordinary least squares estimator, generalized least squares estimator, best linear unbiased estimator.

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