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**DECOMPOSING MATRICES  
WITH JERZY K. BAKSALARY**

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**Abstract**

In this paper we comment on some papers written by Jerzy K. Baksalary. In particular, we draw attention to the development process of some specific research ideas and papers now that some time, more than 15 years, has gone after their publication.

**Keywords:** BLUE, BLUE's covariance matrix, canonical correlations, generalized inverse, linear model, linear sufficiency, OLSE, orthogonal projector, residuals.

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