

LATTICES OF RELATIVE COLOUR-FAMILIES AND ANTIVARIETIES

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Abstract

We consider general properties of lattices of relative colour-families and antivarieties. Several results generalise the corresponding assertions about colour-families of undirected loopless graphs, see [1]. Conditions are indicated under which relative colour-families form a lattice. We prove that such a lattice is distributive. In the class of lattices of antivarieties of relation structures of finite signature, we distinguish the most complicated (universal) objects. Meet decompositions in lattices of colour-families are considered. A criterion is found for existence of irredundant meet decompositions. A connection is found between meet decompositions and bases for anti-identities.

Keywords: colour-family, antivariety, lattice of antivarieties, meet decomposition, basis for anti-identities.

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