

MAXIMAL CLONES AND MAXIMAL PERMUTATION GROUPS

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In memoriam Professor Kazimierz Głazek

Abstract

A fundamental result in universal algebra is the theorem of Rosenberg describing the maximal subclones in the clone of all operations over a finite set. In group theory, the maximal subgroups of the symmetric groups are classified by the O’Nan–Scott Theorem. We shall explore the similarities and differences between these two analogous major results. In addition, we show that a primitive permutation group of diagonal type can be maximal in the symmetric group only if its socle is the direct product of two isomorphic simple groups, because if the number of simple factors of the socle is greater than two, then the group is contained in the alternating group.

Keywords: maximal clones, Rosenberg’s primality criterion, O’Nan–Scott theorem, primitive permutation groups.

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