

Discussiones Mathematicae
General Algebra and Applications 27 (2007) 245–262
doi:10.7151/dmgaa.1129

Nd-SOLID VARIETIES

KLAUS DENECKE

Universität Potsdam, Fachbereich Mathematik
Postfach 601553, 14415 Potsdam, Germany
e-mail: kdenecke@rz.uni-potsdam.de

AND

PRISANA GLUBUDOM

Chiangmai University, Department of Mathematics
Chiangmai, Thailand 50200
e-mail: puprisana@yahoo.com

To the memory of Professor Kazimierz Głazek

Abstract

A non-deterministic hypersubstitution maps any operation symbol of a tree language of type τ to a set of trees of the same type, i.e. to a tree language. Non-deterministic hypersubstitutions can be extended to mappings which map tree languages to tree languages preserving the arities. We define the application of a non-deterministic hypersubstitution to an algebra of type τ and obtain a class of derived algebras. Non-deterministic hypersubstitutions can also be applied to equations of type τ . Formally, we obtain two closure operators which turn out to form a conjugate pair of completely additive closure operators. This allows us to use the theory of conjugate pairs of additive closure operators for a characterization of M -solid non-deterministic varieties of algebras. As an application we consider M -solid non-deterministic varieties of semigroups.

Keywords: Non-deterministic hypersubstitution, conjugate pair of additive closure operators, M -solid non-deterministic variety.

2000 Mathematics Subject Classification: 08A35, 08A40, 08A70.

REFERENCES

- [1] K. Denecke and S.L. Wismath, *Universal Algebra and Applications in Theoretical Computer Science*, Chapman & Hall/CRC, Boca Raton, London, New York, Washington, D.C. 2002.
- [2] K. Denecke, P. Glubudom and J. Koppitz, *Power Clones and Non-Deterministic Hypersubstitutions*, preprint 2005.
- [3] F. Gécseg and M. Steinby, *Tree Languages*, pp. 1–68 in: *Handbook of Formal Languages*, Vol. 3, Chapter 1, Tree Languages, Springer-Verlag 1997.
- [4] K. Denecke and J. Koppitz, *M-solid Varieties of Algebras*, *Advances in Mathematics*, Vol. 10, Springer 2006.
- [5] S. Leeratanavalee, *Weak hypersubstitutions*, Thesis, University of Potsdam 2002.
- [6] K. Menger, *The algebra of functions: past, present, future*, *Rend. Mat.* **20** (1961), 409–430.
- [7] B.M. Schein, and V.S. Trokhimenko, *Algebras of multiplace functions*, *Semigroup Forum* **17** (1979), 1–64.
- [8] W. Taylor, *Abstract Clone Theory*, *Algebras and Orders*, Kluwer Academic Publishers, Dordrecht, Boston, London (1993), 507–530.

Received 30 May 2006

Revised 15 June 2007