

REMARKS TO GŁAZEK'S RESULTS ON n -ARY GROUPS

WIESŁAW A. DUDEK

Institute of Mathematics and Computer Science
Wrocław University of Technology
Wybrzeże Wyspiańskiego 27, 50–370 Wrocław, Poland
e-mail: dudek@im.pwr.wroc.pl

Abstract

This is a survey of the results obtained by K. Głazek and his co-workers. We restrict our attention to the problems of axiomatizations of n -ary groups, classes of n -ary groups, properties of skew elements and homomorphisms induced by skew elements, constructions of covering groups, classifications and representations of n -ary groups. Some new results are added too.

Key words and phrases: polyadic group, n -ary group, variety, representation.

2000 Mathematics Subject Classification: 20N15.

REFERENCES

- [1] V.A. Artamonov, *On Schreier varieties of n -groups and n -semigroups*, (Russian), Trudy Semin. im. I.G. Petrovskogo **5** (1979), 193–203.
- [2] V.D. Belousov and Z. Stojaković, *On infinitary quasigroups*, Publ. Inst. Math. (Beograd) **15** (29) (1973), 31–42.
- [3] A. Borowiec, W.A. Dudek and S. Duplij, *Basic concepts of ternary Hopf algebras*, J. Kharkov National University, ser. Nuclei, Particles and Fields **529** (2001), 21–29.
- [4] A. Borowiec, W.A. Dudek and S. Duplij, *Bi-element representations of ternary groups*, Commun. Algebra **34** (2006), 1651–1670.

- [5] R. Carlsson, *Cohomology of associative triple systems*, Proc. Amer. Math. Soc. **60** (1976), 1–7.
- [6] N. Celakoski, *On some axioms system for n -groups*, Mat. Bilten (Skopje) **1** (1977), 5–14.
- [7] W. Dörnte, *Untersuchungen über einen verallgemeinerten Gruppenbegriff*, Math. Z. **29** (1928), 1–19.
- [8] I.M. Dudek and W.A. Dudek, *On skew elements in n -ary groups*, Demonstratio Math. **14** (1981), 827–833.
- [9] W.A. Dudek, *Remarks on n -groups*, Demonstratio Math. **13** (1980), 165–181.
- [10] W.A. Dudek, *Autodistributive n -groups*, Commentationes Math. Annales Soc. Math. Polonae, Prace Matematyczne **23** (1983), 1–11.
- [11] W.A. Dudek, *Medial n -groups and skew elements*, pp. 55–80 in: “*Proceedings of the V Universal Algebra Symposium ‘Universal and Applied Algebra’, Turawa 1988*”, World Scientific, Singapore 1989.
- [12] W.A. Dudek, *On n -ary groups with only one skew element*, Radovi Matematički **6** (1990), 171–175.
- [13] W.A. Dudek, *Varieties of polyadic groups*, Filomat **9** (1995), 657–674.
- [14] W.A. Dudek, *Idempotents in n -ary semigroups*, Southeast Asian Bull. Math. **25** (2001), 97–104.
- [15] W.A. Dudek, *On some old and new problems in n -ary groups*, Quasigroups and Related Systems **8** (2001), 15–36.
- [16] W.A. Dudek and K. Głazek, *Around the Hosszú-Gluskin Theorem for n -ary groups*, Discrete Math. (2007), (in print).
- [17] W.A. Dudek, K. Głazek and B. Gleichgewicht, *A note on the axioms of n -groups*, Colloquia Math. Soc. J. Bolyai ”Universal Algebra”, (Esztergom (Hungary), North-Holland (Amsterdam 1982)) **29** (1977), 195–202.
- [18] W.A. Dudek and I. Groździńska, *On ideals in regular n -semigroups*, Matematički Bilten (Skopje) **3/4 (29/30)** (1979–1980), 35–44.
- [19] W.A. Dudek and J. Michalski, *On a generalization of Hosszú Theorem*, Demonstratio Math. **15** (1982), 783–805.
- [20] W.A. Dudek and J. Michalski, *On retracts of polyadic groups*, Demonstratio Math. **17** (1984), 281–301.
- [21] W.A. Dudek and J. Michalski, *On a generalization of a theorem of Timm*, Demonstratio Math. **18** (1985), 869–883.

- [22] A.M. Gal'mak, *Translations of n -ary groups*, (Russian), Dokl. Akad. Nauk BSSR **30** (1986), 677–680.
- [23] A.M. Gal'mak, *New axiomatics of an n -ary group*, (Russian), pp. 31–38 in: *Problems in algebra and applied mathematics*, Gomel 1995.
- [24] A.M. Gal'mak, *Polyadic analogs of the Cayley and Birkhof theorems*, Russ. Math. **45** (2001), 10–15 (translation from Izv. Vyssh. Uchebn. Zaved., Mat. No. 2 (2001), 13–18).
- [25] A.M. Gal'mak, *Generalized morphisms of abelian n -ary groups*, Discussiones Math., General Algebra Appl. **21** (2001), 47–55.
- [26] A.M. Gal'mak, *An n -ary subgroup of identities*, (Russian), Vestsi Nats. Akad. Navuk Belarussi Ser. Fiz.-Mat. Navuk **2** (2003), 25–30.
- [27] K. Głazek, *On a construction of covering group for an n -ary group*, Acta Fac. Rer. natur. Univ. Comenian. Math. special Number (1975), 5–8.
- [28] K. Głazek, *Bibliography of n -groups (polyadic groups) and some group-like n -ary sysetms, "n-Ary Structures"*, Proc. Symp., Skopje 1982, 153–289.
- [29] K. Głazek, *Remarks on polyadic groups and their weak homomorphisms*, Contributions on General Algebra **2**, Proc. Klagenfurt Conf. 1982, 133–154.
- [30] K. Głazek, *Algebras of algebraic operations and morphisms of algebraic systems*, (Polish), Acta Univ. Wratislaviensis **1602**, Wrocław Univ. Press 1994.
- [31] K. Głazek and B. Gleichgewicht, *On one method of construction of a covering group* (Russian), Acta Univ. Wratislaviensis No. 188 (1973), 117–123.
- [32] K. Głazek and B. Gleichgewicht, *Abelian n -groups*, Colloquia Math. Soc. J. Bolyai "Universal Algebra" (Esztergom (Hungary), North-Holland (Amsterdam 1982)) **29** (1977), 321–329.
- [33] K. Głazek and B. Gleichgewicht, *On 3-groups and 3-groups polynomially-derived from integral domains*, Semigroup Forum **32** (1985), 61–70.
- [34] K. Głazek and J. Michalski, *On homomorphisms and isomorphisms of term-derived polyadic groups*, pp. 95–102 in: "Proceedings of the II International Symposium, 'n-Ary Structures', Varna 1983".
- [35] K. Głazek and J. Michalski, *On polyadic groups which are term derived from groups*, Studia Sci. Math. Hungar. **19** (1984), 307–315.
- [36] K. Głazek and J. Michalski, *Note on polyadic groups on sets with at most 7 elements*, Beitrage Algebra und Geom. **24** (1987), 151–158.
- [37] K. Głazek and J. Michalski, *Polyadic group operations on small sets*, pp. 85–93 in: "Proceedings Int. Con. 'General Algebra', Krems 1988" (Elsevier, Amsterdam 1990).

- [38] K. Głazek, J. Michalski and I. Sierocki, *On evaluation of numbers of some polyadic groups*, Contributions to General Algebra **3** (1985), 157–171.
- [39] B. Gleichgewicht and K. Głazek, *Remarks on n -groups as abstract algebras*, Colloq. Math. **17** (1967), 209–219.
- [40] B. Gleichgewicht, M.B. Wanke-Jakubowska and M.E. Wanke-Jerie, *On representations of cyclic n -groups*, Demonstratio Math. **16** (1983), 357–365.
- [41] L.M. Gluskin, *Positional operatives*, (Russian), Mat. Sbornik **68** (1965), 444–472.
- [42] M. Hosszú, *On the explicit form of n -group operations*, Publ. Math., Debrecen, **10** (1963), 88–92.
- [43] E. Kasner, *An extension of the group concept* (reported by L.G. Weld), Bull. Amer. Math. Soc. **10** (1904), 290–291.
- [44] Z. Madevski, B. Trpenovski and G. Cupona, *On infinitary associative operations* (in Makedonian), Bull.Soc. Math. Phys. Macedoine **15** (1964), 19–22.
- [45] J. Michalski, *Covering k -groups of n -groups*, Archivum Math. (Brno) **17** (1981), 207–226.
- [46] J.D. Monk and F.M. Sioson, *On the general theory of m -groups*, Fund. Math. **72** (1971), 233–244.
- [47] J. Płonka, *On splitting-automorphism of algebras*, Bull. Soc. Royale Sci. Liege **42** (1973), 303–306.
- [48] A.P. Pojidaev, *Enveloping algebras of Filippov algebras*, Commun. Algebra **31** (2003), 883–900.
- [49] E.L. Post, *Polyadic groups*, Trans. Amer. Math. Soc. **48** (1940), 208–350.
- [50] S.A. Rusakov, *On the definition of n -ary groups*, (Russian), Doklady Akad. Nauk BSSR **23** (1979), 965–967.
- [51] S.A. Rusakov, *A criterion on the existency of n -ary groups*, (Russian), in *Subgroup structure of finite groups*, Navuka i Tekhnika, Minsk (1981), 77–82.
- [52] N.A. Shchuchkin, *Skew endomorphisms on n -ary groups*, Quasigroups and Related Systems **14** (2006), 209–218.
- [53] F.M. Sokhatsky, *On Dudek's problems on the skew operation in polyadic groups*, East Asian Math. J. **19** (2003), 63–71.
- [54] E.I. Sokolov, *On the Gluskin-Hosszú theorem for Dörnte n -groups*, (Russian), Mat. Issled. **39** (1976), 187–189.
- [55] J. Timm, *Kommutative n -Gruppen*, Dissertation, Hamburg 1967.

- [56] V.I. Tyutin, *About the axiomatics of n -ary groups*, (Russian), Doklady Akad. Nauk BSSR **29** (1985), 691–693.
- [57] J. Ušan, *n -groups in the light of the neutral operations*, Math. Moravica, special issue (2003), 3–162.
- [58] L. Vainerman and R. Kerner, *On special classes of n -algebras*, J. Math. Phys. **37** (1996), 2553–2565.
- [59] M.B. Wanke-Jakubowska and M.E. Wanke-Jerie, *On orders of skew elements in finite n -groups*, Demonstratio Math. **12** (1979), 247–253, erratum: **14** (1981), 527–528.
- [60] M.B. Wanke-Jakubowska and M.E. Wanke-Jerie, *On representation of n -groups*, Commentationes Math. Annales Soc. Math. Polonae, Prace Matematyczne **24** (1984), 335–341.
- [61] B. Zeković, *Frobenius n -group algebras*, Discuss. Math. Gen. Algebra Appl. **22** (2002), 153–159.
- [62] B. Zeković and V.A. Artamonov, *On two problems for n -group rings*, Math. Montisnigri **15** (2002), 79–85.

Received 1 May 2006

Revised 25 July 2006