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SUFFICIENT CONDITIONS FOR THE STRONG CONSISTENCY OF LEAST SQUARES ESTIMATOR WITH α-STABLE ERRORS

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Abstract

Let $Y_i = \mathbf{x}_i^T \boldsymbol{\beta} + e_i$, $1 \leqslant i \leqslant n$, $n \geqslant 1$ be a linear regression model and suppose that the random errors $e_1, e_2, \ldots$ are independent and α-stable. In this paper, we obtain sufficient conditions for the strong consistency of the least squares estimator $\widetilde{\boldsymbol{\beta}}$ of $\boldsymbol{\beta}$ under additional assumptions on the non-random sequence $\mathbf{x}_1, \mathbf{x}_2, \ldots$ of real vectors.

Keywords: linear models, least squares estimator, strong consistency, stability.

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References

[1] B.D.O. Anderson and J.B. Moore, *On martingales and least squares linear system identification*, Technical report EE7522 (1975).

[2] B.D.O. Anderson and J.B. Moore, *A matrix Kronecker lemma*, Linear Algebra and its Applications **15** (1976), 227–234.

[3] P. Billingsley, *Probability and Measure*, (third edition) John Wiley & Sons 1995.

[4] Y.S. Chow and H. Teicher, *Probability Theory: Independence*, Interchangeability, Martingales, Springer 1997.

[5] H. Drygas, *Consistency of the least squares and Gauss-Markov estimators in regression models*, Z. Wahrscheinlichkeitstheorie Verw. Gebiete **17** (1971), 309–326.

[6] H. Drygas, *Weak and strong consistency of the least squares estimators in regression model*, Z. Wahrscheinlichkeitstheorie Verw. Gebiete **34** (1976), 119–127.

[7] W. Feller, *An Introduction to Probability Theory and Its Applications - Volume I*, (third edition) John Wiley & Sons 1968.

[8] W. Feller, *An Introduction to Probability Theory and Its Applications - Volume II*, (second edition) John Wiley & Sons 1971.

[9] I.A. Ibragimov and Yu.V. Linnik, *Independent and Stationary Sequences of Random Variables*, Wolters-Noordhoff, Groningen (1971).

[10] M. Jin, *Some new results of the strong consistency of multiple regression coefficients*, p. 514–519 in: *"Proceedings of the Second Asian Mathematical Conference 1995"* (Tangmanee, S. & Schulz, E. eds.), World Scientific.

[11] M. Jin and X. Chen, *Strong consistency of least squares estimate in multiple regression when the error variance is infinite*, Stat. Sin. **9** (1) (1999), 289–296.

[12] T.L. Lai, H. Robbins and C.Z. Wei, *Strong consistency of least squares estimates in multiple regression*, Proc. Natl. Acad. Sci. USA **75** (7) (1978), 3034–3036.

[13] T.L. Lai, H. Robbins and C.Z. Wei, *Strong consistency of least squares estimates in multiple regression II*, J. Multivariate Anal. **9** (1979), 343–362.

[14] J.T. Mexia, P. Corte Real, M.L. Esquível and J. Lita da Silva, *Convergência do estimador dos mínimos quadrados em modelos lineares*, Estaística Jubilar, Actas do XII Congresso da Sociedade Portuguesa de Estatística, Edições SPE (2005), 455–466.

[15] J.T. Mexia and J. Lita da Silva, *Least squares estimator consistency: a geometric approach*, Discussiones Mathematicae - Probability and Statistics **26** (1) (2006), 19–45.

[16] G. Samorodnitsky and M.S. Taqqu, *Stable non-Gaussian Random Processes: Stochastic Models with Infinite Variance*, Chapman & Hall 1994.

[17] V.V. Uchaikin and V.M. Zolotarev, *Chance and Stability, Stable Distributions and Their Applications*, Ultrech 1999.

[18] V.M. Zolotarev, *One-Dimensional Stable Distributions*, American Mathematical Society, Providence, R.I. 1986.

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