

**SYSTEMS OF DIFFERENTIAL INCLUSIONS
IN THE ABSENCE OF MAXIMUM PRINCIPLES
AND GROWTH CONDITIONS**

CHRISTOPHER C. TISDELL*

School of Mathematics

The University of New South Wales

Sydney 2052, Australia

e-mail: cct@maths.unsw.edu.au

Abstract

This article investigates the existence of solutions to second-order boundary value problems (BVPs) for systems of ordinary differential inclusions. The boundary conditions may involve two or more points. Some new inequalities are presented that guarantee *a priori* bounds on solutions to the differential inclusion under consideration. These *a priori* bound results are then applied, in conjunction with appropriate topological methods, to prove some new existence theorems for solutions to systems of BVPs for differential inclusions. The new conditions allow the treatment of systems of BVPs in the absence of maximum principles and growth conditions. The results are also new for differential equations involving Carathéodory or even continuous right-hand sides.

Keywords: boundary value problem, systems of differential inclusions, existence of solutions, *a priori* bounds, two-point boundary conditions, three-point boundary conditions.

2000 Mathematics Subject Classification: 34B10, 34B15.

*C.C. Tisdell gratefully acknowledges the research support of the Australian Research Council's Discovery Projects (DP0450752).

REFERENCES

- [1] J. Andres and L. Górniewicz, Topological fixed point principles for boundary value problems. *Topological Fixed Point Theory and Its Applications*, 1, Kluwer Academic Publishers, Dordrecht 2003.
- [2] A. Arara, M. Benchohra, S.K. Ntouyas and A. Ouahab, *Existence results for boundary value problems for fourth-order differential inclusions with nonconvex valued right hand side*, Arch. Math. (Brno) **40** (3) (2004), 219–227.
- [3] J.P. Aubin and A. Cellina, *Differential inclusions. Set-valued maps and viability theory*, Fundamental Principles of Mathematical Sciences, 264, Springer-Verlag, Berlin 1984.
- [4] E.P. Avgerinos, N.S. Papageorgiou and N. Yannakakis, *Periodic solutions for second order differential inclusions with nonconvex and unbounded multifunction*, Acta Math. Hungar. **83** (4) (1999), 303–314.
- [5] R. Bader, B.D. Gel'man, M. Kamenskii and V. Obukhovskii, *On the topological dimension of the solutions sets for some classes of operator and differential inclusions*, Discuss. Math. Differ. Incl. Control Optim. **22** (1) (2002), 17–32.
- [6] M. Benchohra and S.K. Ntouyas, *Multi point boundary value problems for second order differential inclusions*, Mat. Vesnik **53** (1–2) (2001), 51–58.
- [7] M. Benchohra and S.K. Ntouyas, *On three and four point boundary value problems for second order differential inclusions*, Math. Notes (Miskolc) **2** (2) (2001), 93–101.
- [8] M. Benchohra and S.K. Ntouyas, *Multi-point boundary value problems for lower semicontinuous differential inclusions*, Math. Notes (Miskolc) **3** (2) (2002), 91–99.
- [9] A. Boucherif and B. Chanane, *Boundary value problems for second order differential inclusions*, Int. J. Differ. Equ. Appl. **7** (2) (2003), 147–151.
- [10] A. Boucherif and B. Chanane, *Second order multivalued boundary value problems*, Comm. Appl. Nonlinear Anal. **11** (1) (2004), 85–91.
- [11] Y. Daido, M. Ikehata and G. Nakamura, *Reconstruction of inclusions for the inverse boundary value problem with mixed type boundary condition*, Appl. Anal. **83** (2) (2004), 109–124.
- [12] B.C. Dhage, *On boundary value problems of second order differential inclusions*, Discuss. Math. Differ. Incl. Control Optim. **24** (2004), 73–96.
- [13] T. Donchev and M. Quincampoix, *A two point boundary value problem for a class of differential inclusions*, J. Nonlinear Convex Anal. **5** (1) (2004), 59–69.

- [14] L.H. Erbe and W. Krawcewicz, Boundary value problems for second order nonlinear differential inclusions. *Qualitative theory of differential equations* (Szeged, 1988), 163–171, Colloq. Math. Soc. Janos Bolyai, 53, North-Holland, Amsterdam 1990.
- [15] L.H. Erbe and W. Krawcewicz, Boundary value problems for differential inclusions. *Differential equations* (Colorado Springs, CO, 1989), 115–135, Lecture Notes in Pure and Appl. Math., 127, Dekker, New York 1991.
- [16] L.H. Erbe and W. Krawcewicz, *Nonlinear boundary value problems for differential inclusions $y'' \in F(t, y, y')$* , Ann. Polon. Math. **54** (3) (1991), 195–226.
- [17] L. Erbe, R. Ma and C.C. Tisdell, *On two point boundary value problems for second order differential inclusions*, Dynam. Systems Appl. **15** (1) (2006), 79–88.
- [18] L. Erbe, C.C. Tisdell and P.J.Y. Wong, *On Systems of boundary value problems for differential inclusions*, Acta Math. Sinica (in press).
- [19] M. Filippakis, L. Gasiński and N.S. Papageorgiou, Positive solutions for second order multivalued boundary value problems, *Nonlinear analysis and applications: to V. Lakshmikantham on his 80th birthday*, Vol. 1, 2, 531–547, Kluwer Acad. Publ., Dordrecht 2003.
- [20] L. Gasiński and N.S. Papageorgiou, *Strongly nonlinear multivalued boundary value problems*, Nonlinear Anal. **52** (4) (2003), 1219–1238.
- [21] L. Gasiński and N.S. Papageorgiou, *Nonlinear second-order multivalued boundary value problems*, Proc. Indian Acad. Sci. Math. Sci. **113** (3) (2003), 293–319.
- [22] Y.E. Gliklikh and A.V. Obukhovskii, *On a two-point boundary value problem for second-order differential inclusions on Riemannian manifolds*, Abstr. Appl. Anal. (2003), no. 10, 591–600.
- [23] A.M. Gomaa, *On the solution sets of three-point boundary value problems for nonconvex differential inclusions*, J. Egyptian Math. Soc. **12** (2) (2004), 97–107.
- [24] A. Granas and J. Dugundji, *Fixed point theory*, Springer Monographs in Mathematics, Springer-Verlag, New York 2003.
- [25] N. Halidias and N.S. Papageorgiou, *Existence and relaxation results for nonlinear second-order multivalued boundary value problems in R^N* , J. Differential Equations **147** (1) (1998), 123–154.
- [26] D.A. Kandilakis and N.S. Papageorgiou, *Existence theorems for nonlinear boundary value problems for second order differential inclusions*, J. Differential Equations **132** (1) (1996), 107–125.

- [27] M. Kisielewicz, *Differential inclusions and optimal control*, Mathematics and its Applications (East European Series), 44. Kluwer Academic Publishers Group, Dordrecht; PWN—Polish Scientific Publishers, Warsaw 1991.
- [28] T. Pruszko, *Some applications of the topological degree theory to multivalued boundary value problems*, *Dissertationes Math. (Rozprawy Mat.)* **229** (1984).
- [29] S. Sędziwy, *Fredholm alternative and boundary value problems*, *Proc. Amer. Math. Soc.* **132** (6) (2004), 1779–1784.
- [30] G.V. Smirnov, *Introduction to the theory of differential inclusions*, Graduate Studies in Mathematics, 41, American Mathematical Society, Providence, RI 2002.
- [31] A.N. Vityuk, *Solvability of a three-point boundary value problem for a second-order differential inclusion*, (Russian) *Ukraĭn. Mat. Zh.* **55** (1) (2003), 132–137; translation in *Ukrainian Math. J.* **55** (1) (2003), 164–170.

Received 9 September 2005