

ON GEOMETRY OF THE SET OF ADMISSIBLE
QUADRATIC ESTIMATORS OF QUADRATIC
FUNCTIONS OF NORMAL PARAMETERS

KONRAD NEUMANN AND STEFAN ZONTEK

Faculty of Mathematics, Computer Science and Econometrics
University of Zielona Góra

prof. Z. Szafrana 4a, 65–516 Zielona Góra, Poland

e-mail: k.neumann@wmie.uz.zgora.pl

e-mail: s.zontek@wmie.uz.zgora.pl

Abstract

We consider the problem of admissible quadratic estimation of a linear function of μ^2 and σ^2 in n dimensional normal model $N(K\mu, \sigma^2 I_n)$ under quadratic risk function. After reducing this problem to admissible estimation of a linear function of two quadratic forms, the set of admissible estimators are characterized by giving formulae on the boundary of the set $\mathcal{D} \subset \mathcal{R}^2$ of components of the two quadratic forms constituting the set of admissible estimators. Different shapes and topological properties of the set \mathcal{D} are studied.

Keywords: linear estimator, quadratic estimator, Bayesian quadratic estimator, quadratic loss function, admissibility, quadratic subspace.

2000 Mathematics Subject Classification: 62F10, 62J10.

REFERENCES

- [1] S. Gnot, E. Rafajłowicz and A. Urbańska-Motyka, *Statistical inference in a linear model for spatially located sensors and random input*, Ann. Inst. Statist. Math. **53. 2** (2001), 370–379.

- [2] S. Gnot and J. Kleffe, *Quadratic estimation in mixed linear models with two variance components*, J. Statist. Plann. Inference **8** (1983), 267–279.
- [3] D.A. Harville, *Quadratic unbiased estimation of two variance components for the one-way classification*, Biometrika **56** (1969), 313–326.
- [4] L.R. LaMotte, *Admissibility in linear model*, Ann. Statist. **19** (1982), 245–256.
- [5] L.R. LaMotte, *Admissibility, unbiasedness, and nonnegativity in the balanced, random, one-way anova model*, Linear statistical inference (Poznań, 1984), Lecture Notes in Statist. **35** (1985), 184–199.
- [6] K. Neumann and S. Zontek, *On geometry of the set of admissible invariant quadratic estimators in balanced two variance components model*, Statistical Papers **45** (2004), 67–80.
- [7] A.L. Rukhin, *Quadratic estimators of quadratic functions of normal parameters*, J. Statist. Plann. Inference **15** (1987), 301–310.
- [8] A.L. Rukhin, *Admissible polynomial estimates for quadratic polynomials of normal parameters* (in russian), Zap. Nauchn. Sem. Leningrad. Otdel. Mat. Inst. Steklov. **184**, Issued. Mat. Statist. **9** (1990), 234–247.
- [9] R. Zmyślony, *Quadratic admissible estimators*, (in polish) Roczniki Polskiego Towarzystwa Matematycznego, Seria III: Matematyka Stosowana VII, (1976), 117–122.
- [10] S. Zontek, *Admissibility of limits of the unique locally best linear estimators with application to variance components models*, Probab. Math. Statist. **9. 2** (1988), 29–44.

Received 10 November 2005
Revised 20 October 2006