

## VOLTERRA INTEGRAL INCLUSIONS VIA HENSTOCK-KURZWEIL-PETTIS INTEGRAL

BIANCA SATCO

*Université de Bretagne Occidentale, UFR Sciences et Techniques*

*Laboratoire de Mathématiques CNRS-UMR 6205*

*6 Avenue Victor Le Gorgeu, CS 93837, 29283 BREST Cedex 3, FRANCE*

**e-mail:** bianca.satco@univ-brest.fr

### Abstract

In this paper, we prove the existence of continuous solutions of a Volterra integral inclusion involving the Henstock-Kurzweil-Pettis integral. Since this kind of integral is more general than the Bochner, Pettis and Henstock integrals, our result extends many of the results previously obtained in the single-valued setting or in the set-valued case.

**Keywords:** Volterra integral inclusion, Henstock-Kurzweil integral, Henstock-Kurzweil-Pettis integral, set-valued integral.

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