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**THE METHOD OF UPPER AND LOWER SOLUTIONS
FOR PERTURBED n^{th} ORDER DIFFERENTIAL
INCLUSIONS**

BUPURAO C. DHAGE

Kasubai, Gurukul Colony

Ahmedpur-413 515, Dist: Latur, Maharashtra, India

e-mail: bcd20012001@yahoo.co.in

AND

ADRIAN PETRUȘEL

Department of Applied Mathematics

Babeș-Bolyai University Cluj-Napoca

Kogălniceanu 1, 3400 Cluj-Napoca, Romania

e-mail: petrusel@math.ubbcluj.ro

Abstract

In this paper, an existence theorem for n^{th} order perturbed differential inclusion is proved under the mixed Lipschitz and Carathéodory conditions. The existence of extremal solutions is also obtained under certain monotonicity conditions on the multi-functions involved in the inclusion. Our results extend the existence results of Dhage *et al.* [7, 8] and Agarwal *et al.* [1].

Keywords: differential inclusion, method of upper and lower solutions, existence theorem.

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