

SOME ALGEBRAIC FIXED POINT THEOREMS FOR MULTI-VALUED MAPPINGS WITH APPLICATIONS

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Abstract

In this paper, some algebraic fixed point theorems for multi-valued discontinuous operators on ordered spaces are proved. These theorems improve the earlier fixed point theorems of Dhage (1988, 1991) Dhage and Regan (2002) and Heikkilä and Hu (1993) under weaker conditions. The main fixed point theorems are applied to the first order discontinuous differential inclusions for proving the existence of the solutions under certain monotonicity condition of multi-functions.

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