

## SOME REMARKS ON PRÜFER MODULES

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### Abstract

We provide several characterizations and investigate properties of Prüfer modules. In fact, we study the connections of such modules with their endomorphism rings. We also prove that for any Prüfer module  $M$ , the forcing linearity number of  $M$ ,  $\text{fln}(M)$ , belongs to  $\{0, 1\}$ .

**Keywords:** Prüfer modules, Prüfer domains, invertible submodules, duo modules, forcing linearity number.

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