

## ON SOME PROPERTIES OF CHEBYSHEV POLYNOMIALS

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### Abstract

Letting  $T_n$  (resp.  $U_n$ ) be the  $n$ -th Chebyshev polynomials of the first (resp. second) kind, we prove that the sequences  $(X^k T_{n-k})_k$  and  $(X^k U_{n-k})_k$  for  $n - 2 \lfloor n/2 \rfloor \leq k \leq n - \lfloor n/2 \rfloor$  are two basis of the  $\mathbb{Q}$ -vectorial space  $\mathbb{E}_n[X]$  formed by the polynomials of  $\mathbb{Q}[X]$  having the same parity as  $n$  and of degree  $\leq n$ . Also  $T_n$  and  $U_n$  admit remarkableness integer coordinates on each of the two basis.

**Keywords:** Chebyshev polynomials, integer coordinates.

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