

**NECESSARY CONDITIONS FOR LINEAR
NONCOOPERATIVE N -PLAYER DELTA
DIFFERENTIAL GAMES ON TIME SCALES***

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Abstract

We present necessary conditions for linear noncooperative N -player delta dynamic games on an arbitrary time scale. Necessary conditions for an open-loop Nash-equilibrium and for a memoryless perfect state Nash-equilibrium are proved.

Keywords: delta differential games, dynamic games on time scales.

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