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ON RISK RESERVE UNDER DISTRIBUTION CONSTRAINTS

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Abstract

The purpose of this work is a study of the following insurance reserve model:

$$R(t) = \eta + \int_0^t p(s, R(s))ds + \int_0^t \sigma(s, R(s))dW_s - Z(t),\ t \in [0, T],$$

$$P(\eta \geq c) \geq 1 - \epsilon, \epsilon \geq 0.$$

Under viability-type assumptions on a pair (p, σ) the estimation γ with the property: $\inf_{0 \leq t \leq T} P\{R(t) \geq c\} \geq \gamma$ is considered.

Keywords: martingales, stochastic equations, reserve process, Girsanov's theorem, viability.

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1. Introduction

The classical collective risk theory was initiated by Lundberg (1903). In classical theory the insurer's reserve is described by the process:

$$R_t = u + ct - \sum_{i=1}^{C(t)} Z_i,\ t \geq 0.$$

In this model, u describes the initial risk reserve at time $t = 0$ and c is the constant rate of premium income. The number of claims are generated by a Poisson process $\{C(t), t \geq 0\}$, where the claims $Z_1, Z_2, ...$ form a sequence of independent and identicaly distributed random variables (i.i.d. r.v.'s), with a probability distribution function (p.d.f.) G. The above model is not realistic because the cumulative premium income is a linear function of time. On the other hand, the income of an insurer is not deterministic. In reality, there are fluctuations in the number of customers and the claim arrival intensity may depend on time. Moreover, the insurer may invest the surplus. Finally, the classical model is not so realistic since there is no dependence between the income to the company and the level of risk reserve. To model these additional uncertainties Dufresne and Gerber (1991) considered the perturbated compound Poisson risk model, where the perturbation process (added to the original risk reserve) was a Brownian motion. In Petersen (1990) a model can be found, where the premium part depends on current reserve. It has the following form:

$$R_t = u + \int_0^t p(R_s)ds - \sum_{i=1}^{C(t)} Z_i,\ t \geq 0.$$

In the paper, we propose the model assuming that initial reserve u is a given random variable η with the restriction on its distribution: $P\{\eta \geq c\} \geq 1-\epsilon$, for given constants $c \geq 0$ and $\epsilon \in [0, 1)$. Moreover, in our model we allow the second integral term in premium part of the reserve. The aim of the paper is the lower estimation of the probability that reserve R_t will be over a constant level c, for all times t taken from a finite interval $[0, T]$. In our study, we employ so-called weak tangential condition known from the study of viability problems.

2. MODEL AND NOTATIONS

Let $(\Omega, \mathcal{F}, (\mathcal{F}_t), P)$ be a given filtered probability base and let $T_1, T_2, ...$ be a sequence of positive i.i.d. random variables on it. We introduce also the sequence of (nonnegative) claim amounts: $Z_1, Z_2, \ldots$. Next define the counting process $C(t) = \max\{n \geq 1 : \sum_{i=1}^n T_i \leq t\}$, $t > 0$, with $C(0) = 0$ and the total claim amount $Z(t) = \Sigma_{i=1}^{C(t)} Z_i, t > 0$ and $Z(0) = 0$. Assume, that $Z_1, Z_2, ...$ are i.i.d. discrete valued r.v.'s, with p.d.f. G, independent of the process C. For any r.v. Y, by P^Y we denote the distribution of Y under P, on the measurable space $(R, \beta(R))$. For fixed $c \geq 0$,we consider the following (one dimensional) stochastic equation:

$$(1)\qquad \begin{aligned}&R(t)=\eta+\int_0^t p(s,R(s))ds+\int_0^t \sigma(s,R(s))dW_s - Z(t),\ t\in[0,T]\\ &P^{\eta}\{[c,\infty)\}\geq 1-\epsilon.\end{aligned}$$

For a given $\epsilon\in[0,1)$, the initial reserve η is assumed to be $\mathcal{F}_0$ measurable r.v. with restriction on its distribution as above (we do not require η to be a nonnegative r.v.). Integral terms above describe "drift" and "perturbation" parts of the premium income process depending on the reserve R. We assume that W is the Wiener process, adapted to the given, rigth continuous and complete filtration $(\mathcal{F}_t)$. Let $D=\sigma(Z(T))$ be a σ-field generated by r.v. $Z(T)$. Then we can expand the original filtration $(\mathcal{F}_t)$ introducing the new one (say) (Γ_t) by: $\Gamma_t=\cap_{s>t}\sigma(\mathcal{F}_s,D), t\in[0,T]$. We impose the following assumption on the model.

A1. Functions $p,\sigma: R_+\times R\to R$ are assumed to be jointly continuous, such that:

$$|p(t,x)-p(t,y)|+|\sigma(t,x)-\sigma(t,y)|\leq k|x-y|,$$

for some $k>0$, all x,y and $t\in[0,T]$.

The aim of the paper can be described in the following way: the problem is to find a solution to (1), and constant $\gamma\in[0,1)$ with the property:

$$\inf_{t\in[0,T]} P^{R(t)}\{[c,\infty)\}\geq\gamma.$$

In the next section, we shall transform the above formulated problem to the one being a case of one-dimenssional weak viability problem connected with an extended filtration and equivalent probability. We recall now main notions needed in the sequel. Let $\{K_t, t\in[0,T]\}$ be a family of closed subsets of real line. Consider a stochastic equation on some filtered probability space $(\Omega^*,\mathcal{F}^*,(\mathcal{F}^*_t),P^*)$:

$$\begin{aligned}&X(t)=S+\int_0^t p(s,X(s))ds+\int_0^t \sigma(s,X(s))dW_s,\ t\in[0,T],\\ &P^{*S}\{K_o\}\geq\gamma,\ \text{ with }\ \gamma\in(0,1].\end{aligned}$$

Recall that under Lipschitz type assumption above, imposed on the mappings p, σ, there exists a unique (strong) solution X to this equation.

Definition 2.1. By $(\gamma, \{K_t\}, (\Omega^*, \mathcal{F}^*, (\mathcal{F}_t^*), P^*))$-*viable solution to the equation above we mean any solution* X *with the property*: $P^{*X(t)}\{K_t\} \geq \gamma$, *for every* $t \in [0, T]$.

For the Ito stochastic equation or inclusion the viability problem, with $\gamma = 1$ was studied in Aubin and Da Prato (1990, 1998), Kisielewicz (1995), Gauthier and Thibault (1993) and others. The case when $K_t \equiv K$, with K being a fixed nonempty, closed set and $\gamma \in (0, 1)$ (weak viability or viability under distribution constraints) was considered in Michta (1998) and in Mazliak (1999), where a controlled diffusion case was studied. For our purposes we present now sufficient conditions to ensure (γ, K)-viable solution. For the set K, by $\Pi_\gamma(K)$ we denote the set of probability distributions μ, on a real line, such that $\mu(K) \geq \gamma$. It is an easy observation that for every nonempty and closed set K, the set $\Pi_\gamma(K)$ is nonempty, convex and closed under weak convergence. Consequently, for r.v. ξ we set: $\xi \in^{P^*} \Pi_\gamma(K)$ to denote that $P^{*\xi} \in \Pi_\gamma(K)$.

Definition 2.2. $((\gamma, K, (\Omega^*, \mathcal{F}^*, (\mathcal{F}_t^*), P^*))$-weak tangential condition (WTC)). Functions p and σ satisfy $(\gamma, K, (\Omega^*, \mathcal{F}^*, (\mathcal{F}_t^*), P^*))$-WTC with respect to X, if for each $t \in [0, T)$, and every F_t^*-measurable r.v. ξ, such that $\xi \in^{P^*} \Pi_\gamma(K)$, there exist $\epsilon' > 0$, $t' \in (t, t + \epsilon')$ and a sequence of continuous processes $\{U^n, V^n, n \geq 1\}$ on $[t, t']$, for which:

$$\text{(WTC1)} \qquad \xi + \int_t^z U_s^n ds + \int_t^z V_s^n dW_s \in^{P^*} \Pi_\gamma(K), \ \text{ for } \ t \leq z \leq t',$$

$$\text{(WTC2)} \qquad (U^n, V^n) \to^{P^*} (p(X), \sigma(X)),$$

where the symbol $\to^{P^*}$ means the convergence in probability (under P^*) in $C([t, t'], R^2)$.

Remark 2.1. The conditions given in the Definition 2.2 are similar to Gauthier and Thibault‘s tangential condition in the case of the viability when $\gamma = 1$.

3. Associated stochastic equation and equivalent probability under change of filtration

Consider the following (say) associated equation:

$$
\begin{aligned}
&X(t) = \eta + \int_0^t p(s, X(s))ds + \int_0^t \sigma(s, X(s))dW_s - Z(T),\ t \in [0, T], \\
&P^\eta\{[c, \infty)\} \geq 1 - \epsilon.
\end{aligned} \tag{2}
$$

Let us notice that the "new" initial random variable in (2), say $S = \eta - Z(T)$, is an anticipating one with respect to the filtration $(\mathcal{F}_t)$. To make it nonanticipating we use an expanded filtration (Γ_t) introduced before. Simultaneously with the above equation, we consider another one, with respect to (Γ_t). Namely:

$$
\begin{aligned}
&X(t) = S + \int_0^t p(s, X(s))ds + \int_0^t \sigma(s, X(s))dW_s,\ t \in [0, T], \\
&P^S\{[c, \infty)\} \geq \gamma.
\end{aligned} \tag{3}
$$

Here the constant γ (it will be shown later) depends on ϵ, G,and laws of r.v. η and $C(T)$, and has to be calculated under the assumption: $P^\eta\{[c, \infty)\} \geq 1 - \epsilon$. The next important consequence of expanding the original filtration is that the process W need not be a Wiener process with respect to the filtration (Γ_t). Generaly to preserve the property of being a local martingale (with respect to expanded filtration) even as a semimartingale it is generaly also a delicate problem (see e.g. Yor, 1985). But in our lattice-type situation we have:

Lemma 3.1. *Let us suppose that the claim size r.v.'s $Z_1, Z_2, \ldots$ are discrete valued. Then the process W is a semimartingale with respect to the filtration (Γ_t).*

Proof. Let B be a countable set such that $P\{Z_n \in B\} = 1$, for $n \geq 1$. Next, we define the events: $A_{(n, b_1, b_2, \ldots, b_n)} = \{C(T) = n, Z_1 = b_1, Z_2 = b_2, \ldots, Z_n = b_n\}$, $n \geq 1, b_i \in B$, for $1 \leq i \leq n$. Additionaly, for $n = 0$ we set $A_{(0)} = \{C(T) = 0\}$. Let Λ denote the family of all events defined as above. It is easily seen that Λ consists of countable many disjoint events. Thus elements of Λ can be indexed by nonnegative integers, i.e. $\Lambda = \{A_n : n \geq 0\}$.

Moreover, the probability of each such event is positive and $D = \sigma(Z(T)) = \sigma(\Lambda)$. The next observation is that Λ forms a countable partition of Ω. By $P_n(\cdot) = P(\cdot|A_n)$, $n \geq 0$, we define a sequence of probability measures. Then $P_n \ll P$, for each $n \geq 0$. From Protter (1990, Theorem 2, Chapter 2) W is a $((\mathcal{F}_t), P_n)$-semimartingale for every $n \geq 1$. Next, expanding the original filtration $(\mathcal{F}_t)$ by all events with P_n-probability 0 or 1 we get a larger one, say $(\mathcal{F}_t^n)$, such that W is a $((\mathcal{F}_t^n), P_n)$-semimartingale, for $n \geq 1$. Then we have $\mathcal{F}_t \subset \Gamma_t \subset \mathcal{F}_t^n$, for $t \geq 0$, and $n \geq 0$. Since the process W is $\mathcal{F}_t$-adapted, then it is Γ_t-adapted as well. Thus by Stricker's Theorem (see e.g. Protter, 1990) we get that W is also a (Γ_t, P_n)-semimartingale. Since $P(\cdot) = \sum_{n\geq 0} P_n(\cdot)P(A_n)$, then we claim that W is a (Γ_t, P)-semimartingale as well, what completes the proof.

Owing to Lemma 3.1 we assume:

A2. There exists a Γ-predictable process L such that $N_t = W_t - \int_0^t L_s ds$ is a (Γ, P)-continuous local martingale.

A3. $E\{\exp(\frac{1}{2}\int_0^T L_s^2 ds)\} < \infty$ *(Novikov's condition).*

Remark 3.2. The lemma above in some sense justifies the assumption A2. Since the process W is a (Γ_t, P)-semimartingale with continuous paths, then it can be expressed as a sum of (Γ_t, P)-local martingale and a process of finite variation. In fact, the assumption A2 expresses the shape of W.

Recall (see e.g. Protter, 1990) that for the process Y being a continuous semimartingale, by $\langle Y, Y\rangle$ we denote the quadratic variation process of Y, defined by:

$$\langle Y, Y\rangle_t = \lim_{n\to\infty} \sum_{t_j^n \in \pi_n} (Y_{t_{j+1}^n} - Y_{t_j^n})^2,$$

where $\pi_n : 0 = t_0^n \leq t_1^n \leq \ldots \leq t_{k_n}^n = t$ denotes the finite partition of the interval $[0, t]$ such that $\lim_{n\to\infty} \max_{0\leq j\leq k_n-1}(t_{j+1}^n - t_j^n) = 0$, and the convergence is meant in probability. Similarly, for continuous semimartingales X, Y, their quadratic covariation process $\langle X, Y\rangle_t$ is defined by:

$$\langle X, Y\rangle_t = \lim_{n\to\infty} \sum_{t_j^n \in \pi_n} (X_{t_{j+1}^n} - X_{t_j^n})(Y_{t_{j+1}^n} - Y_{t_j^n}).$$

Let us define a (Γ_t, P)-continuous local martingale $Y = -\int L_s dN_s$, where processes L and N have been defined in A.2. Since $\langle Y, Y\rangle_t = \int_0^t L_s^2 d\langle N, N\rangle_s$

and since $\langle N,N\rangle$ computed under (Γ_t,P) is equal to $\langle W,W\rangle$ computed under $(\mathcal{F}_t,P)$ (in fact, quadratic variation does not depend on the filtration), then $\langle Y,Y\rangle_t = \int_0^t L_s^2 ds$. Hence (A3) $E\{\exp(\frac{1}{2}\langle Y,Y\rangle_T)\} < \infty$. Thus by Novikov's Theorem the exponential process $D_t = \exp(Y_t - \frac{1}{2}\langle Y,Y\rangle_t)$ is a (Γ_t,P)-martingale. By Doleans Theorem it is a solution to the equation $D_t = 1 + \int_0^t D_s dY_s$. Hence, it follows, in particular that $E_P D_T = 1$. We define a new probability Q, $dQ = D_T dP$. Then $Q \sim P$. It can be proved (see Protter, 1989) that W is a (Γ_t,Q)-local martingale. Thus the equation (3) can be considered on the enlarged probability space $(\Omega,\mathcal{F},(\Gamma_t),Q)$:

$$
\begin{aligned}
&X(t) = S + \int_0^t p(s,X(s))ds + \int_0^t \sigma(s,X(s))dW_s,\ t\in[0,T],\\
&Q^S\{[c,\infty)\} \geq \gamma.
\end{aligned}
\tag{4}
$$

To obtain the constant γ in the initial distribution constraints of eq. (3) or (4), let $p_n = P\{C(T)=n\} > 0$, for $n=0,1,\ldots$. Recall that for a given $\epsilon\in[0,1)$ equation (2) is considered under constraint $P^\eta\{[c,\infty)\}\geq 1-\epsilon$. Then we can formulate:

Lemma 3.2.

a) $P^{\eta,Z(T)} = Q^{\eta,Z(T)}$. *Consequently*: $P^\eta = Q^\eta, P^S = Q^S$.

b) $P^\eta\{[c,\infty)\} \geq 1-\epsilon \iff P^S\{[c,\infty)\}(= Q^S\{[c,\infty)\}) \geq \gamma$, where $\gamma = p_0(1-\epsilon) + \sum_{n=1}^\infty p_n E[G^{*n}(\eta - c)]$.

Proof. a) Since η, $(T_1,T_2,\ldots,Z_1,Z_2,\ldots)$ and the process (W_t) are independent under P, then particularly η, $Z(T)$ and D_T are P-independent r.v.'s. Hence, for every Borel subsets $A,B\in\beta(R)$ we get: $Q^{\eta,Z(T)}(A\times B) = E_P(D_T I_{(\eta\in A)} I_{(Z(T)\in B)}) = E_P(D_T)E_P(I_{(\eta\in A)}I_{(Z(T)\in B)}) = P^{\eta,Z(T)}(A\times B)$.

For part b) it is easily seen that:

$$P(S\geq c) = p_0 P(\eta\geq c) + \sum_{n=1}^\infty p_n E[G^{*n}(\eta - c)].$$

Thus taking: $\gamma = p_0(1-\epsilon) + \sum_{n=1}^\infty p_n E[G^{*n}(\eta - c)]$ we get: $P^S\{[c,\infty)\}\geq\gamma$, where G^{*n} denotes the n-th convolution of the common p.d.f. G of the i.i.d. r.v.'s $Z_1,Z_2,\ldots$. The converse part is obvious.

Following above remarks we can formulate:

Theorem 3.1. *Let $0 < \epsilon < 1$ be given and let assumtions* A1, A2 *and* A3 *hold. The following are equivalent:*

a) *X is a solution to equation* (2) *(under P), with initial constraint $P^{\eta}\{[c,\infty)\} \geq 1-\epsilon$*

b) *X is a solution to equation* (3) *(under P) with initial constraint $P^{S}\{[c,\infty)\} \geq \gamma$*

c) *X is a solution to equation* (4) *(under Q) with initial constraint $Q^{S}\{[c,\infty)\} \geq \gamma$.*

Moreover, $P^X = Q^X$ on $C([0,T])$.

Proof. Let X be a $(\mathcal{F}_t, P)$-solution to equation (2). Then by calculations above we have that $P^{\eta}\{[c,\infty)\} \geq 1-\epsilon$ if and only if $P^{S}\{[c,\infty)\} \geq \gamma$, and by A2 the process X is also a (Γ_t, P)-solution to the equation:

$$X(t) = S + \int_0^t p(s,X(s))ds + \int_0^t \sigma(s,X(s))dN_s + \int_0^t L_s\sigma(s,X(s))d\langle N,N\rangle_s,$$

$$P^{S}\{[c,\infty)\} \geq \gamma,$$

(5)

with (Γ_t, P)-local martingale N. Since $dQ = D_T dP$ (and $Q \sim P$), then by Girsanov's Theorem (see e.g. Protter, 1990) the process N is a (Γ_t, Q)-semimartingale of the form:

$$N_t = [N_t - \int_0^t \frac{1}{D_s} d\langle D,N\rangle_s] + \int_0^t \frac{1}{D_s} d\langle D,N\rangle_s,$$

with its "(Γ_t, Q)-local martingale" part given in the bracket. Moreover, since $D_t = 1 + \int_0^t D_s dY_s$, and $Y_t = -\int_0^t L_s dN_s$, then

$$\int_0^t \frac{1}{D_s} d\langle D,N\rangle_s = \int_0^t \frac{D_s}{D_s} d\langle Y,N\rangle_s = -\int_0^t L_s d\langle N,N\rangle_s = -\int_0^t L_s ds.$$

Hence we have that equation (5) has the form:

$$X(t) = S + \int_0^t p(s,X(s))ds + \int_0^t \sigma(s,X(s))dW_s, t \in [0,T],$$

with (Γ_t, Q)-local martinagale W. By Lemma 3.2 we have also that $Q^{S}\{[c,\infty)\} \geq \gamma$.

To present the viability result to reserve process R we need the following lemma:

Lemma 3.3. *Let X and R denote the solutions to equations* (4) *and* (1) *respectively. Then $R(t) \geq X(t)$, for $t \in [0,T]$.*

Proof. It follows immediately from the fact $Z(T) \geq Z(t)$, thus $\eta - Z(T) \leq \eta - Z(t)$, for $t \in [0,T]$, and the Proposition 2.18 p. 239 in Karatzas and Shreve (1988).

Theorem 3.2. *Let us assume that conditions* A1, A2 *and* A3 *hold, and let X be a solution to equation* (3). *If functions p and σ satisfy* $(\gamma, [c,\infty), (\Omega, \mathcal{F}, (\Gamma_t), P))$-WTC *with respect to X, then the solution R to equation* (1) *has the following property: $\inf_{t\in[0,T]} P^{R(t)}\{[c,\infty)\} \geq \gamma$.*

Proof. By Theorem 3.1 and Lemma 3.3, it is enough to show that X is a $(\gamma, [c,\infty), (\Omega, \mathcal{F}, (\Gamma_t), Q))$-viable solution to eq. (4). Let us consider the set $A = \{\, a \in [0,T] : X_t \in^Q \Pi_\gamma([c,\infty))$ for all $t \in [0,a]\}$. The idea of the proof is to show that $A = [0,T]$. First, since X is a solution to equation (4), then we claim $0 \in A$. Next, we show that A is a closed subset of the interval $[0,T]$. Indeed, if $a \in clA$, then we can take a sequence $a_n \in A$ such that $a_n \to a$, if $n \to \infty$. Then $X_t \in^Q \Pi_\gamma([c,\infty))$ for $t \leq a_n, n \geq 1$. A continuity of sample paths of the process X implies: $X_{a_n} \to X_a$ Q a.s. Since $Q \sim P$ we have this convergence P a.s. as well. This implies that $X_{a_n} \to^d X_a$ both under Q and P. Thus, since $\{Q^{X_{a_n}}\} \subset \Pi_\gamma([c,\infty))$, then by Theorem 2.1 in Billingsley (1968) we get $Q^{X_a} \in \Pi_\gamma([c,\infty))$. Hence $a \in A$. Finally, let $\theta = \sup A$. To finish the proof it is enough to show that $\theta = T$. Suppose $\theta < T$. Since $\theta \in A$, then $X_\theta \in^Q \Pi_\gamma(K_c)$. By Theorem 3.1 we have $X_\theta \in^P \Pi_\gamma(K_c)$. From the (WTC)-assumption taken with $t = \theta$, and $\xi = X_\theta$, there exist $\epsilon' > 0$, $\theta' \in (\theta, \theta + \epsilon')$, and two sequences $(U^n), (V^n)$ of continuous processes on $[\theta, \theta + \epsilon']$ such that (WTC1) and (WTC2) hold. By (WTC2), and Kurtz and Protter (1991, Theorem 2.2) we get:

$$\int_\theta^{\theta'} U_s^n ds + \int_\theta^{\theta'} V_s^n dN_s + \int_\theta^{\theta'} L_s V_s^n ds \to^P \int_\theta^{\theta'} p(s, X(s)) ds$$
$$+ \int_\theta^{\theta'} \sigma(s, X(s)) dN_s + \int_\theta^{\theta'} L_s \sigma(s, X(s)) ds,$$

with (Γ_t, P)-local martingale N. Since $Q \sim P$, then the same kind of convergence holds under Q as well. But under P :

$$\int_\theta^{\theta'} V_s^n dN_s + \int_\theta^{\theta'} L_s V_s^n ds = \int_\theta^{\theta'} V_s^n dW_s,$$

with (Γ_t, P)-semimartingale W, and then:

$$\xi + \int_\theta^{\theta'} U_s^n ds + \int_\theta^{\theta'} V_s^n dW_s \to^Q X_{\theta'}.$$

Using (WTC1), and once again the fact that the latter convergence implies the convergence in distribution (both under P and Q), we conclude that $Q^{X_{\theta'}} \in \Pi_\gamma([c,\infty))$. Hence $\theta' \in A$, what contradicts the nature of θ.

Example. Let us assume that r.v. $C(T)$ has a geometric distribution: $p_n = \beta(1-\beta)^n, n = 0,1,2,..., 0 < \beta < 1$.

Assume also that $P\{\eta \geq c\} \geq 1 - \epsilon$, for some $\epsilon \in [0,1)$. Then we have:

$$P\{S \geq c\} = (1-\beta)P\{\eta \geq c\} + \beta E\{G * F_{Z(T)}(\eta - c)\}. \tag{6}$$

Hence we can put: $\gamma = (1-\beta)(1-\epsilon) + \beta E\{G * F_{Z(T)}(\eta - c)\}$.

Note that in "the deterministic case", when η is a deterministic constant, $\eta = x \geq 0$ and $c = 0$, equation (6) can be written as: $F_{Z(T)} = (1-\beta)\delta_0 + \beta G * F_{Z(T)}$ (so called *defective renewal equation),* where δ_0 denotes the p.d.f. concentrated at 0.

On the other hand, let $x_0 = \sup\{x : G(x) < 1\}$, and let m_G denote the moment generating function for p.d.f. G, i.e. $m_G(s) = \int_0^\infty \exp(sx)dG(x)$. Assume that there exists a solution $\alpha > 0$ to the equation $m_G(\alpha) = \frac{1}{\beta}$. In Bergmann and Stoyan (1976) it was shown (in particular) that $1 - F_{Z(T)}(x) \leq a\exp(-\alpha x)$, for $x \geq 0$, where $a = \sup_{x\in[0,x_0)} \dfrac{(1-G(x))\exp(\alpha x)}{\int_x^\infty \exp(\alpha y)dG(y)}$.

Having this, we get from equality (6):

$$P\{S \geq c\} \geq (1-\beta)P\{\eta \geq c\} + \beta E\{G * f_{\alpha,a}(\eta - c)\},$$

where $f_{\alpha,a}(x) = (1 - a\exp(-\alpha x))I(x)_{[0,\infty)}$.

In this case: $\gamma = (1-\beta)(1-\epsilon) + \beta E\{G * f_{\alpha,a}(\eta - c)\}$.

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