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ON USEFUL SCHEMA IN SURVIVAL ANALYSIS AFTER HEART ATTACK

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Abstract

Recent model of lifetime after a heart attack involves some integer coefficients. Our goal is to get these coefficients in simple way and transparent form. To this aim we construct a schema according to a rule which combines the ideas used in the Pascal triangle and the generalized Fibonacci and Lucas numbers

Keywords: lifetime after heart attack, distribution, Fibonacci number, Lucas number, Pascal triangle.

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1. Introduction

This note is motivated by a real medical problem. Many doctors believe that a patient survives a heart attack (in medical terminology: myocardial infarction) unless a succeeding attack occurs in a week. The length of this critical period (for definition see [12] p. 372) is still controversial. Treating heart attacks as failures in Bernoulli trials we reduce the lifetime after a heart attack to the waiting time for the first failure followed by a success run shorter than a given k. Its distribution, recently presented in [25], involves some integer coefficients. Our goal is to get these coefficients both in a simple way and in a transparent form. For this aim we construct a schema according to a rule which combines the ideas used in the Pascal triangle and the generalized Fibonacci and Lucas numbers.

Systems of positive integers have been investigated very intensively in the last decade. In fact this area become a specialized branch of discrete mathematics represented, among others, by five books [10, 19, 27, 28] and [15], two specialized journals (Fibonacci Quarterly and Journal of Integer Sequences) and the On-Line Encyclopedia of Integer Sequences [22]. The main attention focused on Fibonacci, Lucas and Catalan Numbers (FNs, LNs and CNs for abbreviation) with possible generalizations (cf. [3, 4, 5, 9, 14, 17, 20, 24, 30] and [7]) on polynomials generated by them (cf. [2, 3, 6, 18, 23, 26] and [1]). A nice geometric interpretation both FNs and LNs within the Pascal triangle (PT) has been revealed by Koshy [16]. In recent works [13] and [29] FNs, LNs and CNs were studied in terms of graph theory.

This note is the next in the series of works [8, 11, 21] and [13] on schemata of positive integers. It is a response to the real need.

2. From distribution of lifetime after heart attack to schema of its coefficients

The waiting time for the first failure followed by a success run shorter than k may be expressed by the following model.

Let $X = (X_1, X_2, X_3, \dots)$ be a sequence of independent identically distributed random variables taking values 1 or 0 with probabilities p and $q = 1 - p$, where 0 is interpreted as a failure. Given a positive integer k called critical period, define a statistic $T = t(X)$ as the minimal integer n such that $X_n = 0$ and, either $n \leq k$ or $X_m = 0$ for some m satisfying the condition $0 < n - m \leq k$. The statistic T is said to be waiting time for the first failure followed by a success run shorter than k.

Recently the probability mass function of the waiting time in this model was derived by Stępniak [25] in the form

$$P(T = n) = \begin{cases} p^n t, & if \;\; n = 1, 2, \dots, k, \\ (n - k - 1)p^n t^2, & if \;\; n = k + 1, \dots, 2k + 1, \\ p^n t^2 \sum\limits_{r=0}^{\left[\frac{n-k-2}{k+1}\right]} a_r t^r, & if \;\; n > 2k + 1, \end{cases}$$

where $t = \frac{q}{p}$, $[x]$ means the integer part of a number x, while a_r, for $r = 0, \dots, \left[\frac{n-k-2}{k+1}\right]$, is the system of the integer coefficients defined by

(1) $$a_r = a_{n,r;k} = \binom{n-(r+1)k-1}{r+1} - \binom{n-(r+2)k-1}{r+1}.$$

It is well known (cf. e.g. [7, 18, 20, 26]) that the generalized FNs and LNs operate with the same recurrence relation

(2) $$N(n) = N(n-1) + N(n-k-1),$$

but with different initial conditions. On the other hand the construction of the Pascal triangle with its entries

(3) $$\binom{n}{r} = \begin{cases} \frac{n!}{r!(n-r)!}, & \text{if } \ 0 \le r \le n \\ 0, & \text{otherwise} \end{cases},$$

is based on the relation

(4) $$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$$

with the initial conditions

$$\binom{n}{r} = \begin{cases} 1, & \text{if } \ r = 0 \\ 0, & \text{if } \ n = 1 \text{ and } r > 0. \end{cases}$$

This construction may be explained by the following schema:

	0		r-1	r				
0	1							...
	1	1						...
	1	2	1					...
n-1	1	3	[3]	[1]				...
n	1	4	6	[4]	1			...
	1	5	10	10	5	1		...
	1	6	15	20	15	6	1	...
	...	...	...	...	...	...	...	...

Combining (2) and (4) one can get a k-step recurrence relation of type

$$\binom{n}{r}_k = \binom{n-1}{r}_k + \binom{n-k-1}{r-1}_k.$$

This relation may be also completed by some initial condition (as in the generalized FNs and LNs).

It appears that a similar rule may be used to the system of the coefficients in distribution of the lifetime after heart attack.

Definition 1. The system of the coefficients $a_{n,r;k}$ defined by the formula (1) for a given integer $k \geq 0$ and for $n, r =, 1, 2, \ldots.$ is said to be k-schema.

Remark 2. For $k = 0$ the k-schema coincides with the usual Pascal triangle.

Theorem 3. *The entries $a_{n,r;k}$ in k-schema satisfy the recurrence relation*

$$a_{n,r;k} = a_{n-1,r;k} + a_{n-k-1,r-1;k} \tag{5}$$

for $n \geq k+2$ and $r \geq 1$ with the initial conditions

$$a_{n,r;k} = \begin{cases} 0, & \text{if } n < k+2 \\ n-k-1, & \text{if } k+2 \leq n < 2k+2 \text{ and } r = 0 \\ k, & \text{if } n \geq 2k+2 \text{ and } r = 0\,. \end{cases}$$

Proof. Let us start from the initial conditions.

By (1) and (3), $a_{r,n;k} = 0$ if and only if $\binom{n-(r+1)k-1}{r+1} = 0$, i.e., when $n - 1 < (r+1)(k+1)$; in particular when $n < k+2$.

If $n \in \{k+2, \ldots, 2k+1\}$ then $n < 2k+2$. In consequence

$$a_{n,0;k} = \binom{n-k-1}{1} - \binom{n-2k-1}{1} = n-k-1.$$

Now let $n \geq 2k+2$. Then

$$a_{n,0;k} = \binom{n-k-1}{1} - \binom{n-2k-1}{1} = n-k-1-(n-2k-1) = k.$$

For relation (5) we shall use the property (4). By definition (1)

$$\begin{aligned} a_{n-1,r;k} &= \binom{n-1-(r+1)k-1}{r+1} - \binom{n-1-(r+2)k-1}{r+1} \\ &= \binom{n-(r+1)k-2}{r+1} - \binom{n-(r+2)k-2}{r+1} \end{aligned}$$

and

$$\begin{aligned} a_{n-k-1,r-1;k} &= \binom{n-k-1-rk-1}{r} - \binom{n-k-1-(r+1)k-1}{r} \\ &= \binom{n-(r+1)k-2}{r} - \binom{n-(r+2)k-2}{r}. \end{aligned}$$

Thus by (1) and (4) we get (5). ■

Theorem 3 constitutes a basis of our schema. For example, if $k = 3$, this schema takes the following form

	0	r-1	r	...
0	0			...
	0			...
	0			...
	0			...
	0			...
	1			...
	2			...
	3			...
	3			...
	3	1		...
	3	3		...
n-k-1	3	6		...
	3	9		...
	3	12	1	...
n-1	3	15	4	...
n	3	18	10	...
...	...	...	...	...

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