Discussiones Mathematicae
Probability and Statistics 26 (2006) 5–17

UNIT ROOT TEST UNDER INNOVATION OUTLIER CONTAMINATION SMALL SAMPLE CASE

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Abstract

The two sided unit root test of a first-order autoregressive model in the presence of an innovation outlier is considered. In this paper, we present three tests; two are usual and one is new. We give formulas computing the size and the power of the three tests when an innovation outlier (IO) occurs at a specified time, say k. Using a comparative study, we show that the new statistic performs better under contamination. A Small sample case is considered only.

Keywords: autoregressive process, Dickey-Fuller test, innovation outlier, power, size.

2000 Mathematics Subject Classification: Primary 62F11; Secondary 62M10.

1. Introduction

Consider a time series $\{x_t\}$ which follows the model

$$(1) \qquad (1-\rho B)x_t = \epsilon_t, \qquad t = \ldots, -1, 0, 1, \ldots, n\,,$$

where $\{\epsilon_t\}_{t=1,\ldots,n}$ is a sequence of independent normally distributed random variables with mean 0 and variance 1, whereas B denotes the backshift operator such that $Bx_t = x_{t-1}$. We assume that $x_0 = 0$ with probability 1.

Suppose that all what we observe is the segment of observations

$$x_1, x_2, \ldots, x_n \tag{2}$$

and we want to test the hypothesis

$$H_0 : \rho = 1 \qquad \text{vs} \qquad H_1 : \rho \neq 1 \tag{3}$$

at the significance level α.

Various authors have treated the problem of unit root test. Dickey and Fuller (1979)wrote a pioneer paper where they proposed their well known Dikey-Fuller statistic. Phillips (1987) and Phillips and Perron (1988) suggested a criterion for correction of the bias in Dickey-Fuller statistic. For more details, see Diebold (1988), Perron (1989) and Sims and Uhlig (1991) and Fuller (1996, Chap. 10). Also, many authors studied the effect of outliers on unit root tests. Franses and Haldrup (1994) showed that, in the case of Dickey-Fuller tests, there is over-rejection of the unit root hypothesis when additive outliers occur. Outliers on unit root tests in AR(1) are investigated by Shin *et al.* (1996). Maddala and Rao (1997) show that, when n goes to infinity, the impacts of finite additive outliers will go to zero. Vogelsang (1999) proposed two robust procedures to detect outliers and adjust the observations.

According to (3), we consider tests of the form

$$\text{Reject } H_0 \text{ if } \hat{\rho}^2 > c,$$

where $\hat{\rho}$ is a suitable statistic (estimator of ρ) and c a constant.

Three statistics are proposed. The first is based on Dickey-Fuller statistic defined by

$$T_{DF} = n(\hat{\rho}_{LS} - 1)\,, \tag{4}$$

where

$$\hat{\rho}_{LS} = \left[\sum_{t=2}^{n} x_t x_{t-1}\right]\left[\sum_{t=2}^{n} x_{t-1}^2\right]^{-1}. \tag{5}$$

Note that $\hat{\rho}_{LS}$ is the well known least squares estimator of ρ.

The second is of the form

$$T_{SYM} = -(n-2)^{1/2}(1+\hat{\rho}_S)^{-1/2}(1-\hat{\rho}_S)^{1/2}\,, \tag{6}$$

where $\hat{\rho}_S$ is the simple symmetrical estimator of ρ defined by

$$\hat{\rho}_S = \frac{\sum_{t=2}^{n} x_{t-1}x_t}{\frac{1}{2}\left(x_1^2 + x_n^2\right) + \sum_{t=2}^{n-1} x_t^2}\,. \tag{7}$$

T_{SYM} is the corresponding t-statistic given by Fuller (1996).

Finally, we propose a new statistic given by the following formula

$$T_{MED} = x_1/MED\{x_2, x_3, \ldots, x_n\}\,, \tag{8}$$

where $MED\{x_2, x_3, \ldots, x_n\}$ means the median of $x_2, x_3, \ldots, x_n$.

In this paper, T_{MED} is called a median statistic. We will show that the size of the usual tests changes slightly if an innovation outlier occurs. However, when the test statistic T_{MED} is used, the size of the test is stable under contamination.

2. The Dickey-Fuller and symmetrical statistics under contamination

Before studying the given statistics under contamination, note that, using easy computations, we can write

$$P\left(T_{DF}^2 > c\right) = 1 - P(1-\sqrt{c}/n < \hat{\rho}_{LS} < 1+\sqrt{c}/n)$$

and

$$P\left(T_{SYM}^2 > c\right) = 1 - P(\hat{\rho}_S > (n-2-c)/(n-2+c))\,.$$

Also, we remark that $\hat{\rho}_{LS}$ and $\hat{\rho}_S$ can be written as a ratio of two quadratic forms. Indeed, let $X = (x_1, x_2, \ldots, x_n)^T$ be the vector of observations. Then we have

$\hat{\rho}_{LS} = (X^T.M_2.X)^{-1}.(X^T.M_1.X)$ with

$$M_1 = \begin{pmatrix} 0 & 1/2 & 0 & \ldots & 0 \\ 1/2 & 0 & 1/2 & \ldots & 0 \\ 0 & 1/2 & 0 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 1/2 \\ 0 & 0 & 0 & 1/2 & 0 \end{pmatrix} \text{ and } M_2 = \begin{pmatrix} 1 & 0 & 0 & \ldots & 0 \\ 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Also, we have $\hat{\rho}_S = (X^T.D_2.X)^{-1}.(X^T.D_1.X)$ with $D_1 = M_1$ and

$$D_2 = \begin{pmatrix} 1/2 & 0 & 0 & \ldots & 0 \\ 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 1 & 0 \\ 0 & 0 & 0 & 0 & 1/2 \end{pmatrix}.$$

Both $\hat{\rho}_{LS}$ and $\hat{\rho}_S$ are of the form $T = (X^T.R_2.X)^{-1}.(X^T.R_1.X)$, where R_1 and R_2 are symmetric. Assume that, at a position $k \in]1, n[$, an outlier of magnitude Δ occurs. Hence, instead of the segment (2), we observe the following observations $z_1, z_2, \ldots, z_n$, where

$$z_t = x_t \quad \forall t < k \quad ; \quad z_k = x_k + \Delta \quad \text{and} \quad z_t = \rho z_{t-1} + \epsilon_t \quad \forall t > k.$$

The process $\{z_t\}$ generated by the contaminant is called the innovation outlier model (IO) introduced by Fox (1972). Assume that $Z = (z_1, z_2, \ldots, z_n)^T$. Then, under IO contamination, we observe $T^* = (Z^T.R_2.Z)^{-1}.(Z^T.R_1.Z)$ instead of $T = (X^T.R_2.X)^{-1}.(X^T.R_1.X)$.

Proposition 1. *For a given $\rho = \rho_0$,*

$$P_{IO}(T^* > c) = 0.5 + \frac{1}{\pi}\int_0^\infty \frac{\sin f^*(u,\Delta)}{ug^*(u,\Delta)}du,$$

where

$$f^*(u,\Delta) = \frac{1}{2}\sum_{i=1}^{n} Arctan(\lambda_i u) + \frac{\Delta^2 u}{2}\sum_{i=1}^{n}\frac{\lambda_i Q_{k,i}^2}{1+\lambda_i^2 u^2} \tag{9}$$

and

$$g^*(u,\Delta) = \prod_{i=1}^{n}\left(1+\lambda_i^2 u^2\right)^{1/4} . Exp\left\{\frac{\Delta^2 u^2}{2}\sum_{i=1}^{n}\frac{\lambda_i^2 Q_{k,i}^2}{1+\lambda_i^2 u^2}\right\}, \tag{10}$$

where $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigenvalues of the matrix $B = A^T.(R_1 - c.R_2).A$. $Q_{k,i}$ is the (k,i) element of the orthogonal matrix Q containing the normalized eigenvectors of B. The matrix A is defined by

$$A = \begin{pmatrix} 1 & 0 & 0 & \ldots & 0 \\ \rho_0 & 1 & 0 & \ldots & 0 \\ \rho_0^2 & \rho_0 & 1 & \ldots & \vdots \\ \vdots & \vdots & \ldots & \ddots & 0 \\ \rho_0^{n-1} & \rho_0^{n-2} & \ldots & \rho_0 & 1 \end{pmatrix}$$

an $n \times n$-matrix generated by the coefficient ρ_0.

Proof. First, one can write $Z = AV$ with $V^T = (v_1, v_2, \ldots, v_n)$ such that

$$v_t = \varepsilon_t \quad \forall t \neq k \quad \text{and} \quad v_k = \varepsilon_k + \Delta,$$

$$P_{IO}(T^* > c) = P_{IO}(V^T.B.V > 0) \qquad \text{with} \qquad B = A^T.(R_1 - c.R_2).A\ .$$

Due to the fact that B is a nonsingular symmetric matrix with real and distinct eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$, we can find an orthogonal matrix $Q = (Q_{ij})_{i,j=1,\ldots,n}$ $(Q^{-1} = Q^T)$ such that

$$\Lambda = Q^{-1}.B.Q$$

is a diagonal matrix with diagonal elements $\lambda_1, \lambda_2, \ldots, \lambda_n$.

$$\begin{aligned}
P_{IO}(T^* > 0) &= P_{IO}(V^T.B.V > 0) \\
&= P_{IO}(V^T.Q.\Lambda.Q^{-1}.V > 0) \\
&= P_{IO}((Q^T V)^T.\Lambda.(Q^T.V) > 0) \\
&= P_{IO}\left(\lambda_1 \left(\sum_{i=1}^{n} Q_{i1} v_i\right)^2 + \ldots + \left(\sum_{i=1}^{n} Q_{in} v_i\right)^2 > 0\right) \\
&= P_{IO}\left(\lambda_1 Y_1 + \ldots + \lambda_n Y_n > 0\right),
\end{aligned}$$

where $Y_j = \sum_{i=1}^{n} Q_{ij} v_i$ $(j = 1, \ldots, n)$ are independent and distributed as $N(Q_{kj}\Delta, 1)$.

Then

$$P_{IO}(V^T.B.V > 0) = P_{IO}\left(\sum_{i=1}^{n} \lambda_j \chi_1^2(\beta_j) > 0\right) = P_{IO}(S > 0),$$

where $\chi_1^2(\beta_j),\ \ j = 1, 2, \ldots, n$ are independent random variables distributed according to chi-square with one degree of freedom and the non-centrality parameter $\beta_j = Q_{kj}^2 \Delta^2$.

According to Imhoff (1961) theorem, since T^* is of the form $T^* = \sum_{i=1}^n \lambda_j \chi_{h_j}^2(\delta_j^2)$, we can write

$$P_{IO}(T^* > c) = 0.5 + \frac{1}{\pi} \int_0^\infty \frac{\sin f^*(u, \Delta)}{u g^*(u, \Delta)} du,$$

where

$$f^*(u, \Delta) = \frac{1}{2} \sum_{i=1}^n h_i . \Big[Arctan(\lambda_i u) + \delta_i^2 \lambda_i u (1 + \lambda_i^2 u^2)^{-1} \Big]$$

and

$$g^*(u, \Delta) = \prod_{i=1}^n (1 + \lambda_i^2 u^2)^{h_i/4} \ . \ Exp \left\{ \frac{1}{2} \sum_{i=1}^n \frac{\delta_j \lambda_i u}{1 + \lambda_i^2 u^2} \right\}.$$

In our case, $h_i = 1$ and $\delta_i = Q_{ki} \Delta$, $\forall i = 1, 2, \ldots, n$. Hence

$$f^*(u, \Delta) = \frac{1}{2} \sum_{i=1}^n Arctan(\lambda_i u) + \frac{\Delta^2 u}{2} \sum_{i=1}^n \frac{\lambda_i Q_{k,i}^2}{1 + \lambda_i^2 u^2}$$

and

$$g^*(u, \Delta) = \prod_{i=1}^n (1 + \lambda_i^2 u^2)^{1/4} \ . \ Exp \left\{ \frac{\Delta^2 u^2}{2} \sum_{i=1}^n \frac{\lambda_i^2 Q_{k,i}^2}{1 + \lambda_i^2 u^2} \right\}.$$

This completes the proof. ■

To obtain the power of the test, we just have to write

$$P_{IO}\Big({T^*_{DF}}^2 > c\Big)$$

$$= 1 - P_{IO}\Big(1 - \sqrt{c}/n < \Big(Z^T.M_2.Z\Big)^{-1}.\Big(Z^T.M_1.Z\Big) < 1 + \sqrt{c}/n\Big)$$

and

$$P_{IO}\Big({T^*_{SYM}}^2 > c\Big)$$

$$= 1 - P_{IO}\Big(\Big(Z^T.D_2.Z\Big)^{-1}.\Big(Z^T.D_1.Z\Big) > \Big(n-2-c\Big)/\Big(n-2+c\Big)\Big)$$

with T^*_{DF} and T^*_{SYM} are the statistics under contamination. The result can be obtained by applying the above proposition.

3. The Median statistic under contamination

Recall that the median statistic is defined here by the formula

$$T_{MED} = x_1/MED\{x_2, x_3, \ldots, x_n\}.$$

Note that

$$P\Big({T_{MED}}^2 > c\Big) = P\Big(-1/\sqrt{c} \leq 1/T_{MED} \geq 1/\sqrt{c}\Big),$$

where c is a critical value such that $P(T^2_{MED} > c) = \alpha$. Our aim is to study the behavior of this probability with respect to Δ when we observe T_{MED} in the contaminated model.

Since analytical treatments are rather complicated , let us give the exact formula of the power test when $n = 3$. Even if a series of length 3 has no practical sense, our aim is to illustrate mathematically the impact of an IO outlier on the T_{MED}-test only.

Proposition 2. *If $n = 3$ and an IO outlier of magnitude Δ occurs at a specified position k, then for a given $\rho = \rho_0$*

$$P_{IO}\Big({T^*_{MED}}^2 > c\Big) = P_{IO}(k_1 < Y_1/Y_2 < k_2),$$

where T^*_{MED} *is the statistic* T_{MED} *observed in the contaminated model ,* Y_1 *and* Y_2 *are independent random variables distributed according to* $N(m_1, 1)$ *and* $N(m_2, 1)$ *respectively with* m_1 *and* m_2 *given by*

$$(m_1, m_2) = \begin{cases} (0, 2\Delta) & \textit{if} \quad k = 1\,, \\ (\rho_0\Delta/2\gamma, 0) & \textit{if} \quad k = 2\,, \\ (\Delta/2\gamma, 0) & \textit{if} \quad k = 3\,, \end{cases}$$

where $\gamma = \sqrt{1 + (1+\rho_0)^2}/2$ *and*

$$k_1 = -\frac{1}{\gamma}\left(\frac{1}{\sqrt{c}} + \frac{\rho_0 + \rho_0^2}{2}\right) \qquad \textit{and} \qquad k_2 = \frac{1}{\gamma}\left(\frac{1}{\sqrt{c}} - \frac{\rho_0 + \rho_0^2}{2}\right).$$

Proof. First, observe that $x_1 = \epsilon_1$ and

$$P_{IO}({T^*_{MED}}^2 > c) = P_{IO}(-1/\sqrt{c} \leq 1/T^*_{MED} \leq 1/\sqrt{c}).$$

Then, using easy calculations, we can write:

If $k = 1$, then $z_1 = x_1 + \Delta$, $z_2 = x_2 + \rho_0\Delta$, $z_3 = x_3 + \rho_0^2\Delta$ and

$$\begin{aligned} \frac{1}{T^*_{MED}} &= \frac{z_2 + z_3}{2z_1} \\ &= \frac{\rho_0 + \rho_0^2}{2} + \frac{(1+\rho_0)\epsilon_2 + \epsilon_3}{2(\epsilon_1 + \delta)} = \frac{\rho_0 + \rho_0^2}{2} + \gamma.\frac{Y_1}{Y_2}\,, \end{aligned}$$

where Y_1 is $N(0, 1)$ and Y_2 is $N(\Delta, 1)$.

If $k = 2$, then $z_1 = x_1$, $z_2 = x_2 + \Delta$, $z_3 = x_3 + \rho\Delta$ and

$$\begin{aligned} \frac{1}{T^*_{MED}} &= \frac{z_2 + z_3}{2z_1} \\ &= \frac{\rho_0 + \rho_0^2}{2} + \frac{(1+\rho_0)\epsilon_2 + \epsilon_3 + \rho_0\Delta}{2\epsilon_1} = \frac{\rho_0 + \rho_0^2}{2} + \gamma.\frac{Y_1}{Y_2}\,, \end{aligned}$$

where Y_1 is $N(\rho_0\Delta/2\gamma, 1)$ and Y_2 is $N(0,1)$.

If $k = 3$, then $z_1 = x_1$, $z_2 = x_2$, $z_3 = x_3 + \Delta$ and

$$\frac{1}{T^*_{MED}} = \frac{z_2 + z_3}{2z_1}$$

$$= \frac{\rho_0 + \rho_0^2}{2} + \frac{(1+\rho_0)\epsilon_2 + \epsilon_3 + \Delta}{2\epsilon_1} = \frac{\rho_0 + \rho_0^2}{2} + \gamma.\frac{Y_1}{Y_2}\,,$$

where Y_1 is $N(\Delta/2\gamma, 1)$ and Y_2 is $N(0,1)$.

Note that, in the three cases, $1/T^*_{MED}$ is written in the form

$$\frac{1}{T^*_{MED}} = \frac{\rho_0 + \rho_0^2}{2} + \gamma.\frac{Y_1}{Y_2}\,.$$

So we obtain

$$P_{IO}\left({T^*_{MED}}^2 > c\right)$$

$$= P_{IO}(-1/\sqrt{c} \leq 1/T^*_{MED} \geq 1/\sqrt{c})$$

$$= P_{IO}\left(k_1 < \frac{Y_1}{Y_2} < k_2\right)$$

with

$$k_1 = -\frac{1}{\gamma}\left(\frac{1}{\sqrt{c}} + \frac{\rho_0 + \rho_0^2}{2}\right) \qquad \text{and} \qquad k_2 = \frac{1}{\gamma}\left(\frac{1}{\sqrt{c}} - \frac{\rho_0 + \rho_0^2}{2}\right)\,.$$

This completes the proof of the proposition. ■

We can add that, following Cabuk and Springer (1990), the probability density function of Y_1/Y_2 for $n = 3$ is given by the formula:

$$h(y) = \left\{ \frac{1}{2\pi} Exp\left[\frac{1}{2}((m_1 y + m_2)^2/(y^2+1) - (m_1^2 + m_2^2))\right]\right\}$$
$$. \left\{\left[\frac{2}{y^2+1} Exp(-(m_1 y + m_2)^2/2(y^2+1))\right]\right.$$
$$\left. + \left[\sqrt{2\pi}(m_1 y + m_2)/(y^2+1)^{3/2}\right]\left[2\Phi\left((m_1 y + m_2)/\sqrt{y^2+1}\right) - 1\right]\right\}$$
$$-\infty < y < \infty,$$

where $\Phi(u) = \frac{1}{2\pi}\int_{-\infty}^{u} e^{-x^2/2}dx$. Using the formula given above for $n = 3$ the power of the test is given by

$$P_{IO}({T^*_{MED}}^2 > c) = \int_{k_1}^{k_2} h(y)dy\,. \tag{11}$$

The formula (11) can be computed using Monte Carlo methods. For example, Table 1 gives the size of the test for $n = 3$ and the position equals $k = 2$. The variance of the estimator of the integral is less than 0.0015. So it shows that the IO outlier has no effect on the size when the magnitude varies slightly.

Table 1. Variation of the size of the test under IO contamination for $n = 3$, $k = 2$ and $\rho_0 = 1.0$

Δ	0.0	0.1	0.3	0.4	0.5	1.0
$P({T^*_{MED}}^2 > c)$	0.05	0.0490	0.0487	0.0486	0.0485	0.0485

4. Power comparison study

In this section, we propose to compare the power of the Dickey-Fuller, the symmetrical and the median statistics in the contaminated model using simulation procedures. Table 2 presents some simulated values of the power according to Dickey-Fuller (T_{DF}), symmetrical (T_{SYM}) and median (T_{MED}) statistics. Since the value of the position k of the contaminant does not play any role, we give results only for some fixed values of the position.

Table 2. Variation of the power of the test with Δ in IO model

n	k	ρ	T	Δ						
				-1.00	-0.50	-0.10	0.00	0.10	0.50	1.00
10	8	0.5	DF	0.001	0.001	0.001	0.001	0.000	0.000	0.000
			SYM	0.000	0.000	0.000	0.000	0.000	0.000	0.000
			MED	0.144	0.162	0.167	0.170	0.173	0.182	0.182
		0.7	DF	0.004	0.004	0.004	0.004	0.004	0.004	0.005
			SYM	0.000	0.000	0.000	0.000	0.000	0.000	0.000
			MED	0.113	0.121	0.131	0.133	0.132	0.132	0.134
		0.9	DF	0.018	0.017	0.017	0.017	0.017	0.018	0.018
			SYM	0.010	0.011	0.011	0.012	0.011	0.010	0.009
			MED	0.078	0.078	0.078	0.077	0.078	0.077	0.077
		1.0	DF	0.054	0.053	0.051	0.052	0.052	0.051	0.055
			SYM	0.040	0.047	0.047	0.048	0.048	0.048	0.041
			MED	0.047	0.048	0.049	0.049	0.049	0.050	0.051
20	15	0.5	DF	0.000	0.000	0.000	0.000	0.000	0.000	0.000
			SYM	0.000	0.000	0.000	0.000	0.000	0.000	0.000
			MED	0.250	0.266	0.272	0.273	0.275	0.282	0.285
		0.7	DF	0.000	0.000	0.000	0.000	0.000	0.000	0.000
			SYM	0.000	0.000	0.000	0.000	0.000	0.000	0.000
			MED	0.178	0.186	0.189	0.190	0.191	0.199	0.198
		0.9	DF	0.007	0.007	0.007	0.007	0.007	0.006	0.005
			SYM	0.000	0.000	0.000	0.000	0.000	0.000	0.000
			MED	0.088	0.088	0.088	0.087	0.086	0.088	0.089
		1.0	DF	0.055	0.053	0.055	0.054	0.053	0.054	0.056
			SYM	0.050	0.053	0.053	0.053	0.053	0.051	0.050
			MED	0.051	0.050	0.050	0.051	0.051	0.051	0.051

Observe that the median statistic presents the highest and the most stable power under original and contaminated model. Also, especially, the size of the median statistic does not change in the presence of the innovation outlier contaminant. Then, one can say that the median statistic proposed in this paper is more stable and robust than the usual well known statistics T_{DF} and T_{SYM} under IO contamination.

Acknowledgements

The authors are thankful to the referee for useful comments.

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Received 11 February 2005