DESCRIBING NEIGHBORHOODS OF 5-VERTICES IN 3-POLYTOPES WITH MINIMUM DEGREE 5 AND WITHOUT VERTICES OF DEGREES FROM 7 TO 11

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Abstract

In 1940, Lebesgue proved that every 3-polytope contains a 5-vertex for which the set of degrees of its neighbors is majorized by one of the following sequences:

\[(6, 6, 7, 7), (6, 6, 6, 7, 9), (6, 6, 6, 6, 11)\],
\[(5, 6, 7, 7, 8), (5, 6, 6, 7, 12), (5, 6, 6, 8, 10), (5, 6, 6, 6, 17), (5, 5, 7, 7, 13), (5, 5, 7, 8, 10), (5, 5, 6, 7, 27), (5, 5, 6, 6, \infty), (5, 5, 6, 8, 15), (5, 5, 6, 9, 11), (5, 5, 5, 7, 41), (5, 5, 5, 8, 23), (5, 5, 5, 9, 17), (5, 5, 5, 10, 14), (5, 5, 5, 11, 13)\].

In this paper we prove that every 3-polytope without vertices of degree from 7 to 11 contains a 5-vertex for which the set of degrees of its neighbors is majorized by one of the following sequences: \( (5, 5, 6, 6, \infty), (5, 6, 6, 6, 15), (6, 6, 6, 6, 6) \), where all parameters are tight.

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1. Introduction

By a 3-polytope we mean a finite 3-dimensional convex polytope. As proved by Steinitz [31], the 3-polytopes are in one to one correspondence with the 3-connected planar graphs.

The degree \( d(v) \) of a vertex \( v \) (\( r(f) \) of a face \( f \)) in a 3-polytope \( P \) is the number of edges incident with it. By \( \Delta \) and \( \delta \) we denote the maximum and minimum vertex degrees of \( P \), respectively. A \( k \)-vertex (\( k \)-face) is a vertex (face) with degree \( k \); a \( k^+ \)-vertex has degree at least \( k \), etc.

The weight of a face in \( P \) is the degree sum of its boundary vertices, and \( w(P) \), or simply \( w \), denotes the minimum weight of \( 5^- \)-faces in \( P \).

In 1904, Wernicke [32] proved that every 3-polytope with \( \delta = 5 \) has a 5-vertex adjacent with a 6\(^-\)-vertex, which was strengthened by Franklin [15] in 1922, who proved that every 3-polytope with \( \delta = 5 \) has a 5-vertex adjacent with two 6\(^-\)-vertices. Recently, Borodin and Ivanova [11] proved that every such 3-polytope has also a vertex of degree at most 6 adjacent to a 5-vertex and another vertex of degree at most 6, which is tight.

We say that \( v \) is a vertex of type \( (k_1, k_2, \ldots) \) or simply a \( (k_1, k_2, \ldots) \)-vertex if the set of degrees of the vertices adjacent to \( v \) is majorized by the vector \( (k_1, k_2, \ldots) \). If the order of neighbors in the type is not important, then we put a line over the corresponding degrees. The following description of the neighborhoods of 5-vertices in a 3-polytope with \( \delta = 5 \) was given by Lebesgue [28, p. 36] in 1940, which includes the results of Wernicke [32] and Franklin [15].

**Theorem 1** (Lebesgue [28]). Every triangulated 3-polytope with minimum degree 5 contains a 5-vertex of one of the following types:

\[
\begin{align*}
(6, 6, 7, 7, 7), & \quad (6, 6, 6, 7, 9), & \quad (6, 6, 6, 6, 11), \\
(5, 6, 7, 7, 8), & \quad (5, 6, 6, 7, 11), & \quad (5, 6, 6, 8, 8), \\
(5, 6, 6, 9, 7), & \quad (5, 7, 6, 6, 12), & \quad (5, 8, 6, 6, 10), & \quad (5, 6, 6, 6, 17), \\
(5, 5, 7, 7, 8), & \quad (5, 13, 5, 7, 7), & \quad (5, 10, 5, 7, 8), \\
(5, 8, 5, 7, 9), & \quad (5, 7, 5, 7, 10), & \quad (5, 7, 5, 8, 8), \\
(5, 5, 7, 6, 12), & \quad (5, 5, 8, 6, 10), & \quad (5, 6, 5, 7, 12), \\
(5, 6, 5, 8, 10), & \quad (5, 17, 5, 6, 7), & \quad (5, 11, 5, 6, 8), \\
(5, 11, 5, 6, 9), & \quad (5, 7, 5, 6, 13), & \quad (5, 8, 5, 6, 11), & \quad (5, 9, 5, 6, 10), & \quad (5, 6, 6, 5, \infty), \\
(5, 5, 7, 5, 41), & \quad (5, 5, 8, 5, 23), & \quad (5, 5, 9, 5, 17), & \quad (5, 5, 10, 5, 14), & \quad (5, 5, 11, 5, 13).
\end{align*}
\]

Theorem 1, along with other ideas in Lebesgue [28], has many applications to plane graph coloring problems (first examples of such applications and a recent survey can be found in [7, 30]). Some parameters of Lebesgue’s Theorem were improved for narrow classes of plane graphs. For example, in 1963, Kotzig [27] proved that every plane triangulation with \( \delta = 5 \) satisfies \( w \leq 18 \) and conjectured
that \( w \leq 17 \). In 1989, Kotzig’s conjecture was confirmed by Borodin [3] in a more general form.

**Theorem 2** (Borodin [3]). Every 3-polytope with \( \delta = 5 \) has a \((5,5,7)\)-face or a \((5,6,6)\)-face, where all parameters are tight.

By a minor \( k \)-star \( S_k^m \) we mean a star with \( k \) rays centered at a 5-vertex. The Lebesgue’s description [28, p.36] of the neighborhoods of 5-vertices in 3-polytopes with minimum degree 5, \( P_5 \), shows that there is a 5-vertex with three 8-vertices. Another corollary of Lebesgue’s description [28] is that \( w(S_3^m) \leq 24 \), which was improved in 1996 by Jendrol’ and Madaras [23] to the sharp bound \( w(S_3^m) \leq 23 \). Furthermore, Jendrol’ and Madaras [23] gave a precise description of minor 3-stars in \( P_5 \): there is a \((6,6,6)\)- or \((5,6,7)\)-star.

Also, Lebesgue [28] proved that \( w(S_4^m) \leq 31 \), which was strengthened by Borodin and Woodall [13] to the sharp bound \( w(S_4^m) \leq 30 \). Note that \( w(S_3^m) \leq 23 \) easily implies \( w(S_2^m) \leq 17 \) and immediately follows from \( w(S_4^m) \leq 30 \) (in both cases, it suffices to delete a vertex of maximum degree from a minor star of minimum weight). In [9], Borodin and Ivanova obtained a tight description of minor 4-stars in \( P_5 \).

As for minor 5-stars in \( P_5 \), it follows from Lebesgue [28, p.36] that if there are no minor \((5,6,6)\)-stars, then \( w(S_5^m) \leq 68 \) and \( h(S_5^m) \leq 41 \). Borodin, Ivanova, and Jensen [10] showed that the presence of minor \((5,5,6,6)\)-stars can make \( w(S_5^m) \) arbitrarily large and otherwise lowered Lebesgue’s bounds to \( w(S_5^m) \leq 55 \) and \( h(S_5^m) \leq 28 \). On the other hand, a construction in [10] shows that \( w(S_5^m) \geq 48 \) and \( h(S_5^m) \geq 20 \). Recently, Borodin and Ivanova [12] proved that \( w(S_5^m) \leq 51 \) and \( h(S_5^m) \leq 23 \).

More results on the structure of edges and higher stars in various classes of 3-polytopes can be found in [1, 2, 4–6, 8, 9, 14, 16, 19–22, 24–26], with a detailed summary in [12].

In [28] Lebesgue did not give a proof of Theorem 1 and only gave its idea. In 2013, Ivanova and Nikiforov [17] gave a full proof of Theorem 1 and corrected the following imprecisions in its statement:

1. in the type \((5,11,5,6,8)\) there should be 15 instead of 11;
2. in the type \((5,17,5,6,7)\) there should be 27 instead of 17;
3. in the type \((6,6,6,6,11)\) the line is not needed;
4. instead of type \((5,6,7,7,8)\) there should be \((5,8,6,7,7)\) and \((5,7,6,8,7)\);
Corollary 4. Every one of the following types: degree 5 confirming the tightness of the type (\(5, 7, 7, 8\)) it suffices to write (\(5, 7, 7, 8\)).

Later on, Ivanova and Nikiforov [18, 29] improved the corrected version of Theorem 1 by replacing 41 and 23 in the types (\(5, 7, 5, 41\)) and (\(5, 5, 8, 5, 23\)) to 31 and 22, respectively.

**Theorem 3** (Ivanova, Nikiforov [17, 18, 29]). Every 3-polytope with minimum degree 5 contains a 5-vertex of one of the following types:

- (6, 6, 6, 7, 7), (6, 6, 6, 7, 9), (6, 6, 6, 6, 8),
- (5, 5, 7, 13), (5, 5, 7, 8, 10), (5, 5, 5, 10, 14), (5, 5, 5, 11, 13).

Theorem 1 subject to the corrections (1)–(6) implies the following fact.

**Corollary 4.** Every 3-polytope with minimum degree 5 contains a 5-vertex of one of the following types:

- (6, 6, 6, 7, 7), (6, 6, 6, 7, 9), (6, 6, 6, 6, 11),
- (5, 5, 5, 7, 13), (5, 5, 5, 8, 23), (5, 5, 5, 9, 17), (5, 5, 5, 10, 14), (5, 5, 5, 11, 13).

We can see already from Theorem 1 that if vertices of degree from 7 to 11 are forbidden, then there is a 5-vertex of one of the following types: (\(5, 5, 5, 6, \infty\)), (\(5, 5, 6, 6, 17\)), (6, 6, 6, 6, 6).

The purpose of this note is to obtain a precise description of 5-stars in this subclass of \(P_5\).

**Theorem 5.** Every 3-polytope with minimum degree 5 and without vertices of degree from 7 to 11 contains a 5-vertex of one of the following types: (\(5, 5, 5, 6, \infty\)), (\(5, 6, 6, 6, 15\)), (6, 6, 6, 6, 6), where all parameters are tight.

2. **Proving Theorem 5**

All parameters in Theorem 5 are best possible. Indeed, the following construction confirming the tightness of the type (\(5, 5, 5, 6, \infty\)) appears in [10]. Take three
concentric $n$-cycles $C^i = v_1^i \cdots v_n^i$, where $n$ is not limited and $1 \leq i \leq 3$, and join $C^2$ with $C^1$ by edges $v_j^2 v_j^1$ and $v_j^2 v_{j+1}^1$, where $1 \leq j \leq n$ (addition modulo $n$). Then do the same with $C^2$ and $C^3$. Finally, join all vertices of $C^1$ with a new $n$-vertex, and do the same for $C^3$.

The tightness of $(6, 6, 6, 6, 6)$ is confirmed by putting a 5-vertex in each face of the dodecahedron.

To confirm the tightness of $(5, 6, 6, 6, 15)$, we take the dodecahedron and insert the fragment shown in Figure 1 into each face. As a result, we have a 3-polytope with only $(5, 6, 6, 6, 15)$-vertices.

Figure 1. The insert in each face of the dodecahedron to produce a 3-polytope with 5-vertices only of type $(5, 6, 6, 6, 15)$.

Now suppose a 3-polytope $P'$ is a counterexample to Theorem 5. Let $P$ be a counterexample on the same number of vertices with maximum possible number of edges.

Remark 6. In $P$, each $4^+$-face $f = v_1 \cdots v_{d(f)}$ with $d(v_1) = 5$ or $d(v_1) \geq 15$ satisfies $d(v_i) \geq 6$ whenever $3 \leq i \leq d(f) - 1$. Otherwise, we could put a diagonal $v_1v_i$, which contradicts the maximality of $P$.

Corollary 7. In $P$, each $4^+$-face has at most two vertices with degree 5 and/or at least 15. Moreover, if there are precisely two such vertices, then they are adjacent to each other.
2.1. Discharging

The sets of vertices, edges, and faces of $P$ are denoted by $V$, $E$, and $F$, respectively. Euler’s formula $|V| - |E| + |F| = 2$ for $P$ implies

$$
\sum_{v \in V} (d(v) - 6) + \sum_{f \in F} (2r(f) - 6) = -12.
$$

(1)

We assign an initial charge $\mu(v) = d(v) - 6$ to every vertex $v$ and $\mu(f) = 2d(f) - 6$ to every face $f$, so that only $5^-$-vertices have negative charge. Using the properties of $P$ as a counterexample, we define a local redistribution of charges, preserving their sum, such that the new charge $\mu'(x)$ is non-negative whenever $x \in V \cup F$. This will contradict the fact that the sum of the new charges is, by (1), equal to $-12$. The technique of discharging is often used in solving structural and coloring problems on plane graphs.

Let $v_1, \ldots, v_{d(v)}$ denote the neighbors of a vertex $v$ in a cyclic order round $v$, and let $f_1, \ldots, f_{d(v)}$ be the faces incident with $v$ in the same order.

We use the following rules of discharging (see Figure 2).

**R1.** Every $4^+$-face gives 1 to every incident $5$-vertex.

**R2.** Every $12^+$-vertex $v$ gives a simplicial $5$-vertex $v_2$ the following charge through a face $f = v_2vv_3$:

(a) $\frac{1}{4}$ if $d(v_3) = 5$,

(b) $\frac{1}{4}$ if $d(v_3) = 6$,

(c) $\frac{1}{4}$ if $d(v_3) = 7$,

(d) $\frac{1}{4}$ if $d(v_3) = 8$.

**R3.** Every $6^+$-vertex gives a $12^+$-vertex $v_2$ the following charge through a face $f = v_2vv_3$:

(a) $\frac{1}{4}$ if $d(v_3) = 5$,

(b) $\frac{1}{4}$ if $d(v_3) = 6$.

Figure 2. Rules of discharging.
(b) \( \frac{1}{2} \) if \( d(v_3) \geq 6 \),
with the following exception.

e If \( d(v) \geq 16, d(v_1) = 5, d(v_3) = d(x) = d(y) = 6 \), where \( v_2 \) is incident to
face \( v_2xy \), then \( v \) gives \( \frac{2}{3} \) to \( v_2 \) through face \( v_2v_3 \) and \( \frac{1}{2} \) through face \( v_1v_2 \).

R3. Suppose a simplicial 5-vertex \( v \) is adjacent to a 16-vertex \( v_1 \), simplicial 5-
vertices \( v_2 \) and \( v_5 \), and \( v_2 \) is surrounded by \( v_1, v, v_3, x, y \), where \( d(v_3) = d(x) = d(y) = 6 \), (consequently \( d(v_4) \geq 12 \)), while \( v_5 \) is surrounded by \( v_1, v, v_4, w, z \),
where \( d(z) \geq 6 \). Then \( v \) gives \( \frac{1}{4} \) to \( v_1 \).

2.2. Proving \( \mu'(x) \geq 0 \) whenever \( x \in V \cup F \)

First consider a face \( f \) in \( P \). If \( d(f) = 3 \), then \( f \) does not participate in
discharging, and so \( \mu'(v) = \mu(f) = 2 \times 3 - 6 = 0 \). Note that every \( 4^+ \)-face is inci-
dent with at most two 5-vertices due to Corollary 7, which implies that \( \mu'(v) = 2d(f) - 6 - 2 \times 1 \geq 0 \) by R1.

Now let \( v \) be a vertex in \( P \).

Case 1. \( d(v) = 5 \). If \( v \) is incident with a \( 4^+ \)-face, then \( \mu'(v) \geq 5 - 6 + 1 = 0 \)
due to R1. In what follows we can assume that \( v \) is simplicial.

Subcase 1.1. \( v \) is incident only with \( 6^+ \)-vertices. Then there is at least one \( v_i \\
with \( d(v_i) \geq 12 \) due to the absence of \( (6,6,6,6,6) \)-vertices in \( P \). Hence, \( \mu'(v) \geq -1 + 2 \times \frac{1}{2} = 0 \) by R2(b).

Subcase 1.2. \( v \) is incident with precisely one 5-vertex. Since there is no \( (5,6,6,6,15) \)-vertex in \( P \), we can assume that \( v \) has either at least two \( 12^+ \)-neighbors,
or precisely one \( 16^+ \)-neighbor. So we have either \( \mu'(v) \geq -1 + 2 \times \frac{1}{2} + 2 \times \frac{1}{4} > 0 \)
by R2(a),(b), or \( \mu'(v) = -1 + \frac{3}{2} + \frac{1}{4} = 0 \) by R2(e), respectively.

Subcase 1.3. \( v \) is incident with at least two 5-vertices. Note that now R2(e)
is not applicable to \( v \). Also note that \( v \) cannot be incident with more than three
5-vertices due to the absence of \( (5,5,6,6,\infty) \)-vertices in \( P \), which implies that \( v \)
has at least two \( 12^+ \)-neighbors. If \( v \) is incident with precisely three 5-vertices,
then we have \( \mu'(v) \geq -1 + 4 \times \frac{1}{4} = 0 \) by R2(a),(b).

Suppose \( v \) is incident with precisely two 5-vertices. If \( v \) does not participate
in R3, then \( \mu'(v) \geq -1 + 3 \times \frac{1}{4} + \frac{1}{2} > 0 \) by R2(a),(b). Note that if \( v \) participates in
R3, then it gives \( \frac{1}{4} \) only to one 16-neighbor, hence \( \mu'(v) \geq -1 + 3 \times \frac{1}{4} + \frac{1}{2} - \frac{1}{4} = 0 \).

Case 2. \( d(v) = 6 \). Since \( v \) does not participate in discharging, we have
\( \mu'(v) = \mu'(v) = 6 - 6 = 0 \).

Case 3. \( 12 \leq d(v) \leq 15 \). Now R2(e) is not applicable to \( v \), so \( v \) sends at most
\( \frac{d(v) - 12}{2} \) through each face by R2(a),(b), which implies that \( \mu'(v) \geq d(v) - 6 - d(v) \times \frac{1}{2} = \\
\frac{d(v) - 12}{2} \geq 0 \).
Case 4. 16 ≤ d(v) ≤ 17. Note that v gives at most $\frac{2}{3}$ through each 3-face and only to a simplicial 5-vertex. If v gives nothing through at least one incident face, then $\mu'(v) \geq 16 - 6 - 15 \times \frac{2}{3} = 0$ by R1, R2. Further, we can assume that v is simplicial and each face takes away some positive charge from v, which implies that each face at v is incident with a 5-vertex, and all 5-vertices adjacent to v are simplicial. Thus, $\mu'(v) \geq d(v) - 6 - d(v) \times \frac{2}{3} = \frac{d(v) - 18}{3}$, and we have the deficiency $\frac{1}{3}$ for a 17-vertex and $\frac{2}{3}$ for a 16-vertex with respect to donating $\frac{2}{3}$ per face.

Suppose $S_k = v_1, \ldots, v_k$ is a sequence of neighbors of v with $d(v_1) \geq 6$, $d(v_k) \geq 6$, while $d(v_i) = 5$ whenever $2 \leq i \leq k-1$ and $k \geq 3$, and $f_1, \ldots, f_{k-1}$ are the corresponding faces. (It is not excluded that $S_k = S_{d(v)}$, which happens when v has precisely one $6^\pm$-neighbor.) We say that the sequence of faces $f_1, \ldots, f_{k-1}$ saves $\varepsilon$ with respect to the level of $\frac{2}{3}$ if these faces take away the total of $(k-1) \times \frac{2}{3} - \varepsilon$ from v.

Remark 8. Only $v_2$ and $v_{k-1}$ in $S_k$ can receive the charge $\frac{2}{3}$ from v by R2(e), while each of the other 5-vertices $v_i$ receives precisely $\frac{1}{3}$ from v through each incident face. So, if $k \geq 5$, then $v_2$ receives at most 1, and $v_3$ receives $\frac{1}{2}$ from v through incident faces.

Remark 9. If v is completely surrounded by 5-vertices, then $\mu'(v) \geq d(v) - 6 - \frac{d(v)}{3} = \frac{d(v) - 12}{3} > 0$, and hence we can assume from now on that the neighborhood of v is partitioned into $S_k$s.

(P1) If $k = 3$, then $\varepsilon = \frac{1}{3}$. Indeed, here $v_2$ receives $\frac{1}{2}$ through each of the faces $v_1v_2$ and $v_2v_3$ by R2(b), whence $\varepsilon = 2 \times \frac{2}{3} - 2 \times \frac{1}{2} = \frac{1}{3}$.

(P2) If $k = 4$, then $\varepsilon = 0$. Now each of $v_2$ and $v_3$ receives at most 1 from v by Remark 8, so $\varepsilon = 3 \times \frac{2}{3} - 2 = 0$.

(P3) If $k = 5$, then $\varepsilon = \frac{2}{3}$. Suppose $w_1, \ldots, w_4$ are the neighbors of $v_1, \ldots, v_5$ such that there are the faces $v_iw_{i+1}$, where $1 \leq i \leq 4$.

If $v_2$ receives 1 by R2(e), then $d(w_1) = d(w_2) = 6$. Hence, $d(w_4) \geq 12$ due to the absence of a $(5,5,6,6,\infty)$-vertex in $P$, which implies that $v_4$ is adjacent to two $12^\pm$-vertices, whence it receives $\frac{1}{2}$ from v through $f_4$ and $\frac{1}{3}$ through $f_3$. Moreover, $v_3$ gives $\frac{1}{4}$ to v by R3. Hence, $\varepsilon = 4 \times \frac{2}{3} - 1 - \frac{1}{2} - \frac{3}{1} + \frac{1}{4} = \frac{2}{3}$.

If R2(e) is not applicable to v, then $\varepsilon = 4 \times \frac{2}{3} - 4 \times \frac{1}{2} = \frac{2}{3}$.

(P4) If $k = 6$, then $\varepsilon = \frac{1}{3}$. Here, each of $v_2$ and $v_5$ receives at most 1, while each of $v_3$ and $v_4$ receives $\frac{1}{2}$ from v by Remark 8, so $\varepsilon = 5 \times \frac{2}{3} - 2 \times 1 - 2 \times \frac{1}{2} = \frac{1}{3}$.

(P5) If $k = 7$, then $\varepsilon = \frac{1}{2}$. Now we have $\varepsilon = 6 \times \frac{2}{3} - 2 \times 1 - 3 \times \frac{1}{2} = \frac{1}{2}$ by Remark 8.
(P6) If \( k \geq 8 \), then \( \varepsilon \geq \frac{2}{3} \). Now we have \( \varepsilon = (k - 1) \times \frac{2}{3} - 2 \times 1 - (k - 4) \times \frac{1}{2} = \frac{k - 4}{6} \geq \frac{2}{3} \).

If \( d(v) = 17 \), then it suffices to assume that the neighborhood of \( v \) consists of pairs of 5-vertices separated from each other by \( 6^+ \)-vertices by (P1)–(P6) (since otherwise we pay off the deficiency), which is impossible due to the fact that 17 is not divisible by 3.

Suppose that \( d(v) = 16 \) and \( \mu'(v) < 0 \). As follows from (P1)–(P6), the neighborhood of \( v \) can have at most one of the paths \( S_{t+2} \) of \( t \) vertices of degree 5, where \( t \in \{1, 4, 5\} \), while all other vertices are partitioned into pairs of 5-vertices separated from each other by 6-vertices. Indeed, if there are either two paths with \( t \in \{1, 4, 5\} \), or at least one path with \( t = 3 \) or \( t \geq 6 \), then we can pay off the deficiency \( \frac{2}{3} \), a contradiction. But none of these cases is possible due to the divisibility by 3. Namely, if \( t = 1 \) we have \( 16 - 2 = 14 \) faces to be divided into triplets of faces with a sequence \( S_4 \) of neighbors of \( v \) as in (P2), or \( 16 - 5 = 11 \) and \( 16 - 6 = 10 \) faces for \( t = 4 \) and \( t = 5 \), respectively; a contradiction.

Case 6. \( d(v) \geq 18 \). Now \( \mu'(v) \geq d(v) - 6 - d(v) \times \frac{2}{3} = \frac{d(v) - 18}{3} \geq 0 \) by R2.

Thus we have proved \( \mu'(x) \geq 0 \) for every \( x \in V \cup F \), which contradicts (1) and completes the proof of Theorem 5.

References


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