A NOTE ON TOTAL GRAPHS

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Abstract


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1. Results

In this paper, we correct the Theorems 1, 4, 8 and 11, and their corollaries of [1]. There was omitted $t(G)$, i.e., the number of triangles in $G$ or $L(G)$ in Theorem 1 of [1]. The total graph $T(G)$ contains triangles in $G$, $L(G)$ and in the incidence graph. All triangles are numbered in the published paper [1] beside triangles in $G$ or $L(G)$. First, we give corrected version of Theorem 1 of [1] as follows by adding the number of omitted triangles $t(G)$, and its proof is in similar lines as before.
Theorem 1. For any \((p, q)\) graph \(G\),

\[
t[T(G)] = 2t(G) + \frac{1}{2} \sum_{i=1}^{p} \left[ d_{G}^{2}(v_{i}) + 2m_{i}\left( \frac{d_{G}(v_{i})}{3} \right) \right],
\]

where \(m_{i} = 1\) if \(d_{G}(v_{i}) \geq 3\); otherwise \(m_{i} = 0\).

Due to the change in the statement of Theorem 1 of [1], the remaining Theorems 4, 8, 11 and their corollaries of [1] are corrected as follows.

Corollary 2. (a) \(t[T(C_{3})] = 8\) and \(t[T(C_{n})] = 2n\) if \(n > 3\).
(b) For \(n \geq 1\), \(t[T(K_{n})] = \frac{1}{6} \left[(n^{2} - n)(n^{2} - 1)\right].\)

Corollary 3. For \(1 \leq i \leq n\) and \(n \geq 2\), \(t[T(\sqcup_{i=1}^{n} C_{m_{i}})] = \frac{2M}{3}(2n^{2} + 1)\) where \(M = m_{1}m_{2}\cdots m_{n}, m_{i} > 3\).

Theorem 4. Let \(G\) be any \((p, q)\)-graph having \(t(G)\) triangles and \(\delta(G) \geq 2\). Then

\[
t[T(\mu(G))] = 8t(G) + \frac{1}{2} \sum_{i=1}^{p} \left[ 3d_{G}^{2}(v_{i}) + d_{G}^{2}(v_{i}) \right] + \left( \frac{18q + 5p + p^{3}}{6} \right).
\]

Corollary 5. For \(n > 3\), \(t[T(\mu(C_{n})]] = \left(\frac{n^{3} + 107n}{6}\right).\)

Corollary 6. For \(n \geq 3\), \(t[T(\mu(K_{n})]] = \frac{1}{6}(9n^{4} - 15n^{3} + 6n^{2} + 6n).\)

Theorem 7. For any \((p, q)\)-graph \(G\),

\[
t[M(G)] = t(G) + \frac{1}{2} \sum_{i=1}^{p} \left[ d_{G}^{2}(v_{i}) + 2m_{i}\left( \frac{d_{G}(v_{i})}{3} \right) \right] - q,
\]

where \(m_{i} = 1\) if \(d_{G}(v_{i}) \geq 3\); otherwise \(m_{i} = 0\).

Corollary 8. For any \((p, q)\)-graph \(G\),

\[
t[M(\mu(G))] = 4t(G) + \frac{1}{2} \sum_{i=1}^{p} \left[ 3d_{G}^{2}(v_{i}) + d_{G}^{2}(v_{i}) \right] + \frac{p(p^{2} - 1)}{6}.
\]

Theorem 9. For any \((p, q)\)-graph \(G\) with \(p \geq 4\),

\[
t[T(C(G))] = 2m + \frac{1}{6}(p^{4} - 3p^{3} + 5p^{2} - 3p + 12q),
\]

where \(m = t(C(G)).\)

Corollary 10. For \(m, n \geq 3\),

\[
t[T(C(K_{m,n}))] = t[T(K_{m+n})] - mn(m + n - 4).\]
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References


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