CLIQUE PARTS INDEPENDENT OF REMAINDERS

ZDZISLAW SKUPIEŃ

Faculty of Applied Mathematics
University of Mining and Metallurgy AGH
al. Mickiewicza 30, 30–059 Kraków, Poland
e-mail: skupien@uci.agh.edu.pl

Let \( p \) and \( t \) stand for positive integers. Let \( R \) denote an edge subset of size \( |R| = (\binom{p}{2}) \mod t \) in the complete graph \( K_p \). Call \( R \) a remainder (or an edge \( t \)-remainder) in the clique \( K_p \).

**Conjecture L** (L reminds of floor symbol). The floor class \( \lfloor K_p/t \rfloor \) is nonempty. In other words, there exists a graph \( F \) such that, for each edge \( t \)-remainder \( R \) in \( K_p \), \( F \) is a \( t \)th part of \( K_p - R \), i.e., \( F \in \lfloor K_p/t \rfloor \).

Conjecture L implies the following conjecture stated in [2].

**Conjecture L*. For each edge \( t \)-remainder \( R \) in \( K_p \), there is an \( F_R \in (K_p - R)/t =: \lfloor K_p/t \rfloor_R \).

**Theorem L'** (Skupień [2]). There exists an edge \( t \)-remainder \( R \) in \( K_p \) such that the floor class \( \lfloor K_p/t \rfloor_R \) is nonempty.

Plantholt’s theorem [1] on chromatic index is equivalent to the truth of Conjecture L with \( t = p - 1 \) and \( p \) being odd.

Conjecture L can be seen true for many pairs \( p, t \), e.g., if \( t \geq p - 1 \) or \( t \) is small: \( t \leq 5 \). If \( t \) is a constant \( (t \geq 4) \), both Conjectures can be reduced to some values of \( p \) in the interval \( t + 2 \leq p \leq 4t - 5 \).

**References**
