THE 1, 2, 3-CONJECTURE AND 1, 2-CONJECTURE FOR SPARSE GRAPHS

DANIEL W. CRANSTON

Virginia Commonwealth University
Richmond, VA, USA

e-mail: dcranston@vcu.edu

SOGOL JAHANBEKAM\(^1\)

University of Colorado Denver
Denver, CO, USA

e-mail: sogol.jahanbekam@ucdenver.edu

AND

DOUGLAS B. WEST\(^2\)

Zhejiang Normal University, Jinhua, China
and University of Illinois, Urbana, IL, USA

e-mail: west@math.uiuc.edu

Abstract

The 1, 2, 3-Conjecture states that the edges of a graph without isolated edges can be labeled from \{1, 2, 3\} so that the sums of labels at adjacent vertices are distinct. The 1,2-Conjecture states that if vertices also receive labels and the vertex label is added to the sum of its incident edge labels, then adjacent vertices can be distinguished using only \{1, 2\}. We show that various configurations cannot occur in minimal counterexamples to these conjectures. Discharging then confirms the conjectures for graphs with maximum average degree less than \(8/3\). The conjectures are already confirmed for larger families, but the structure theorems and reducibility results are of independent interest.

**Keywords:** 1, 2, 3-Conjecture, 1, 2-Conjecture, reducible configuration, discharging method.

**2010 Mathematics Subject Classification:** 05C15, 05C22, 05C78.

---

\(^1\)Research supported in part by National Science Foundation grant DMS 09-01276.

\(^2\)Research supported in part by National Security Agency grant H98230-10-1-0363.
References


Received 29 March 2013
Revised 13 November 2013
Accepted 13 November 2013