ON EULERIAN IRREGULARITY IN GRAPHS

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Abstract

A closed walk in a connected graph $G$ that contains every edge of $G$ exactly once is an Eulerian circuit. A graph is Eulerian if it contains an Eulerian circuit. It is well known that a connected graph $G$ is Eulerian if and only if every vertex of $G$ is even. An Eulerian walk in a connected graph $G$ is a closed walk that contains every edge of $G$ at least once, while an irregular Eulerian walk in $G$ is an Eulerian walk that encounters no two edges of $G$ the same number of times. The minimum length of an irregular Eulerian walk in $G$ is called the Eulerian irregularity of $G$ and is denoted by $EI(G)$. It is known that if $G$ is a nontrivial connected graph of size $m$, then $\binom{m+1}{2} \leq EI(G) \leq 2^{\binom{m+1}{2}}$. A necessary and sufficient condition has been established for all pairs $k, m$ of positive integers for which there is a nontrivial connected graph $G$ of size $m$ with $EI(G) = k$. A subgraph $F$ in a graph $G$ is an even subgraph of $G$ if every vertex of $F$ is even. We present a formula for the Eulerian irregularity of a graph in terms of the size of certain even subgraph of the graph. Furthermore, Eulerian irregularities are determined for all graphs of cycle rank 2 and all complete bipartite graphs.

Keywords: Eulerian walks, Eulerian irregularity.

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References


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