THE NICHE GRAPHS OF INTERVAL ORDERS

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Abstract

The niche graph of a digraph $D$ is the (simple undirected) graph which has the same vertex set as $D$ and has an edge between two distinct vertices $x$ and $y$ if and only if $N^+_D(x) \cap N^+_D(y) \neq \emptyset$ or $N^-_D(x) \cap N^-_D(y) \neq \emptyset$, where $N^+_D(x)$ (resp. $N^-_D(x)$) is the set of out-neighbors (resp. in-neighbors) of $x$ in $D$. A digraph $D = (V, A)$ is called a semiorder (or a unit interval order) if there exist a real-valued function $f : V \rightarrow \mathbb{R}$ on the set $V$ and a positive real number $\delta \in \mathbb{R}$ such that $(x, y) \in A$ if and only if $f(x) > f(y) + \delta$. A digraph $D = (V, A)$ is called an interval order if there exists an assignment $J$ of a closed real interval $J(x) \subset \mathbb{R}$ to each vertex $x \in V$ such that $(x, y) \in A$ if and only if $\min J(x) > \max J(y)$.

Kim and Roberts characterized the competition graphs of semiorders and interval orders in 2002, and Sano characterized the competition-common enemy graphs of semiorders and interval orders in 2010. In this note, we give characterizations of the niche graphs of semiorders and interval orders.

Keywords: competition graph, niche graph, semiorder, interval order.

2010 Mathematics Subject Classification: 05C75, 05C20, 06A06.

References


Received 2 May 2012
Revised 16 April 2013
Accepted 19 April 2013