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EQUITABLE COLORING AND EQUITABLE CHOOSABILITY OF GRAPHS WITH SMALL MAXIMUM AVERAGE DEGREE [1]

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Abstract

A graph is said to be equitably k-colorable if the vertex set $V(G)$ can be partitioned into k independent subsets $V_1, V_2, \ldots, V_k$ such that $||V_i| - |V_j|| \leq 1$ $(1 \leq i, j \leq k)$. A graph G is equitably k-choosable if, for any given k-uniform list assignment L, G is L-colorable and each color appears on at most $\left\lceil \frac{|V(G)|}{k} \right\rceil$ vertices. In this paper, we prove that if G is a graph such that $mad(G) < 3$, then G is equitably k-colorable and equitably k-choosable where $k \geq \max\{\Delta(G), 4\}$. Moreover, if G is a graph such that $mad(G) < \frac{12}{5}$, then G is equitably k-colorable and equitably k-choosable where $k \geq \max\{\Delta(G), 3\}$.

Keywords: graph coloring, equitable choosability, maximum average degree.

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