

DISTANCE MAGIC CARTESIAN PRODUCTS OF GRAPHS

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Abstract

A distance magic labeling of a graph $G = (V, E)$ with $|V| = n$ is a bijection $\ell : V \rightarrow \{1, \dots, n\}$ such that the weight of every vertex v , computed as the sum of the labels on the vertices in the open neighborhood of v , is a constant.

In this paper, we show that hypercubes with dimension divisible by four are not distance magic. We also provide some positive results by proving necessary and sufficient conditions for the Cartesian product of certain complete multipartite graphs and the cycle on four vertices to be distance magic.

Keywords: distance magic labeling, magic constant, sigma labeling, Cartesian product, hypercube, complete multipartite graph, cycle.

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