

GENERALIZED FRACTIONAL AND CIRCULAR TOTAL COLORINGS OF GRAPHS ¹

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Abstract

Let $\mathcal{P}$ and $\mathcal{Q}$ be additive and hereditary graph properties, $r, s \in \mathbb{N}$, $r \geq s$, and $[\mathbb{Z}_r]^s$ be the set of all s -element subsets of $\mathbb{Z}_r$. An (r, s) -fractional

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$(\mathcal{P}, \mathcal{Q})$ -total coloring of G is an assignment $h : V(G) \cup E(G) \rightarrow [\mathbb{Z}_r]^s$ such that for each $i \in \mathbb{Z}_r$ the following holds: the vertices of G whose color sets contain color i induce a subgraph of G with property $\mathcal{P}$, edges with color sets containing color i induce a subgraph of G with property $\mathcal{Q}$, and the color sets of incident vertices and edges are disjoint. If each vertex and edge of G is colored with a set of s consecutive elements of $\mathbb{Z}_r$ we obtain an (r, s) -circular $(\mathcal{P}, \mathcal{Q})$ -total coloring of G . In this paper we present basic results on (r, s) -fractional/circular $(\mathcal{P}, \mathcal{Q})$ -total colorings. We introduce the fractional and circular $(\mathcal{P}, \mathcal{Q})$ -total chromatic number of a graph and we determine this number for complete graphs and some classes of additive and hereditary properties.

Keywords: graph property, $(\mathcal{P}, \mathcal{Q})$ -total coloring, circular coloring, fractional coloring, fractional $(\mathcal{P}, \mathcal{Q})$ -total chromatic number, circular $(\mathcal{P}, \mathcal{Q})$ -total chromatic number.

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