Discussiones Mathematicae
Graph Theory 34 (2014) 849–855
doi:10.7151/dmgt.1771

Full PDF
DMGT Page

A NOTE ON VERTEX COLORINGS OF PLANE GRAPHS[1]

Igor Fabrici[a]

Jochen Harant[b]

Stanislav Jendrol' [a]

and

Roman Soták[a]

[a] *Institute of Mathematics*
P.J. Šafárik University
Košice, Slovak Republic

[b] *Institut für Mathematik*
Technische Universität
Ilmenau, Germany

Abstract

Given an integer valued weighting of all elements of a 2-connected plane graph G with vertex set V, let $c(v)$ denote the sum of the weight of $v \in V$ and of the weights of all edges and all faces incident with v. This vertex coloring of G is *proper* provided that $c(u) \neq c(v)$ for any two adjacent vertices u and v of G. We show that for every 2-connected plane graph there is such a proper vertex coloring with weights in $\{1, 2, 3\}$. In a special case, the value 3 is improved to 2.

Keywords: plane graph, vertex coloring.

2010 Mathematics Subject Classification: 05C10, 05C15.

References

[1] L. Addario-Berry, K. Dalal, C. McDiarmid, B.A. Reed and A. Thomason, *Vertex-coloring edge-weigtings*, Combinatorica **27** (2007) 1–12. doi:10.1007/s00493-007-0041-6

[1]This work was supported by the Slovak Science and Technology Assistance Agency under the contract No. APVV-0023-10 and by Slovak VEGA grant No. 1/0652/12.

[2] L. Addario-Berry, K. Dalaland and B.A. Reed, *Degree constrainted subgraphs*, Discrete Appl. Math. **156** (2008) 1168–1174.
doi:10.1016/j.dam.2007.05.059

[3] K. Appel and W. Haken, *Every planar map is four-colorable,* I. *Discharging*, Illinois J. Math. **21** (1977) 429–490.

[4] M. Axenovich, J. Harant, J. Przybyło, R. Soták and M. Voigt, *A note on adjacent vertex distinguishing colorings number of graphs*, Electron. J. Combin. (submitted).

[5] M. Bača, S. Jendroľ, M. Miller and J. Ryan, *On irregular total labellings*, Discrete Math. **307** (2007) 1378–1388.
doi:10.1016/j.disc.2005.11.075

[6] T. Bartnicki, B. Bosek, S. Czerwiński, J. Grytczuk, G. Matecki and W. Żelazny, *Additive colorings of planar graphs*, Graphs Combin. **30** (2014) 1087–1098.
doi:10.1007/s00373-013-1331-y

[7] J.A. Bondy and U.S.R. Murty, Graph Theory (Springer, 2008).

[8] G. Chartrand, M.S. Jacobson, L. Lehel, O.R. Oellermann, S. Ruiz and F. Saba, *Irregular networks*, Congr. Numer. **64** (1988) 187–192.

[9] S. Czerwiński, J. Grytczuk and W. Żelazny, *Lucky labelings of graphs*, Inform. Process. Lett. **109** (2009) 1078–1081.
doi:10.1016/j.ipl.2009.05.011

[10] R. Diestel, Graph Theory (Springer, 2000).

[11] A. Frieze, R.J. Gould, M. Karoński and F. Pfender, *On graph irregularity strenght*, J. Graph Theory **41** (2002) 120–137.
doi:10.1002/jgt.10056

[12] S. Jendroľ and P. Šugerek, *A note on face coloring entire weightings of plane graphs*, Discuss. Math. Graph Theory **34** (2014) 421–426.
doi:10.7151/dmgt.1738

[13] T.R. Jensen and B. Toft, Graph Coloring Problems (Wiley, 1995).

[14] M. Kalkowski, *A note on* $1,2$*-conjecture*, Electron. J. Combin. (to appear).

[15] M. Kalkowski, M. Karoński and F. Pfender, *Vertex-coloring edge-weightings: towards the* 1-2-3*-conjecture*, J. Combin. Theory (B) **100** (2010) 347–349.
doi:10.1016/j.jctb.2009.06.002

[16] M. Karoński and T. Łuczak, A. Thomason, *Edge weights and vertex colors*, J. Combin. Theory (B) **91** (2004) 151–157.
doi:10.1016/j.jctb.2003.12.001

[17] J. Przybyło and M. Woźniak, *On* $1,2$ *conjecture*, Discrete Math. Theor. Comput. Sci. **12** (2010) 101–108.

[18] T. Wang and Q. Yu, *On vertex-coloring* 13*-edge-weighting*, Front. Math. China **3** (2008) 1–7.
doi:10.1007/s11461-008-0009-8

[19] W. Wang and X. Zhu, *Entire coloring of plane graphs*, J. Combin. Theory (B) **101** (2011) 490–501.
doi:10.1016/j.jctb.2011.02.006

Received 7 May 2013
Revised 13 January 2014
Accepted 13 January 2014