

## STRONG CHROMATIC INDEX OF PLANAR GRAPHS WITH LARGE GIRTH<sup>1</sup>

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### Abstract

Let  $\Delta \geq 4$  be an integer. In this note, we prove that every planar graph with maximum degree  $\Delta$  and girth at least  $10\Delta + 46$  is strong  $(2\Delta - 1)$ -edge-colorable, that is best possible (in terms of number of colors) as soon as  $G$  contains two adjacent vertices of degree  $\Delta$ . This improves [6] when  $\Delta \geq 6$ .

**Keywords:** planar graphs, edge coloring, 2-distance coloring, strong edge-coloring.

**2010 Mathematics Subject Classification:** 05C15.

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Received 5 April 2013  
Revised 30 October 2013  
Accepted 30 October 2013