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EXISTENCE RESULTS FOR ϕ-LAPLACIAN DIRICHLET BVP OF DIFFERENTIAL INCLUSIONS WITH APPLICATION TO CONTROL THEORY

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Abstract

In this paper, we study ϕ-Laplacian problems for differential inclusions with Dirichlet boundary conditions. We prove the existence of solutions under both convexity and nonconvexity conditions on the multivalued right-hand side. The nonlinearity satisfies either a Nagumo-type growth condition or an integrably boundedness one. The proofs rely on the Bonhnenblust-Karlin fixed point theorem and the Bressan-Colombo selection theorem respectively. Two applications to a problem from control theory are provided.

Keywords and phrases: differential inclusions, boundary value problem, fixed point, compact, convex, nonconvex, decomposable, continuous selection, controllability.

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