

UNITARY INVERTIBLE GRAPHS OF FINITE RINGS

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Abstract

Let R be a finite commutative ring with unity. In this paper, we consider set of additive and mutual additive inverses of group units of R and obtain interrelations between them. In general $\varphi(Z_n)$ is even, however we demonstrate that $\varphi(R)$ is odd for any finite commutative ring with unity of $\text{Char}(R) \neq 2$. Further, we present unitary invertible graph related with self and mutual additive inverses of group units. At long last, we establish a formula for counting the total number of basic and non-basic triangles in the unitary invertible graph.

Keywords: finite commutative rings, additive and mutual additive inverses, Euler-function, unitary invertible graphs, basic and non-basic triangles.

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