

ON COALGEBRAS AND TYPE TRANSFORMATIONS

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Abstract

We show that for an arbitrary *Set*-endofunctor T the generalized membership function given by a sub-cartesian transformation μ from T to the filter functor \mathbb{F} can be alternatively defined by the collection of subcoalgebras of constant T -coalgebras. Sub-natural transformations ε between any two functors S and T are shown to be sub-cartesian if and only if they respect μ . The class of T -coalgebras whose structure map factors through ε is shown to be a covariety if ε is a natural and sub-cartesian mono-transformation.

Keywords: coalgebra, endofunctor, filter functor, cartesian transformation, crisp.

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